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To cite this article: N N Aleksandrova *et al* 2019 *J. Phys.: Conf. Ser.* **1203** 012001

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On stress/strain state in a rotating disk

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Abstract. In the framework of mechanics of continuum bodies, the problem of stress/strain state in a high-speed rotating disk of constant thickness has been considered. The material of the disk is assumed to be homogeneous, elastic/perfectly-plastic. In the plastic zone, the stresses and plastic strains are related by some associated law similar to the one employed in deformation theory of plasticity. The general algorithm of the solution covers any smooth plasticity function. At some steps of the algorithm, it is possible to get analytical expressions, particularly, for the quadratic Mises yield criterion. For the given model, the notion of control parameters (external and internal) has been introduced. The allowable boundaries of external parameters have been defined as well. For some states of the disk, the coherent values of external parameters have been obtained. The results are represented graphically to show various states of the disk. The usage of piecewise plasticity functions has been briefly discussed. The results obtained can be used in preliminary engineering design and related numerical codes.

Introduction

Analytical solutions for rotating disk problems are common topics in classical books devoted to elasticity [1] and plasticity [2, 3] theories as well as the problem has been reflected in numerous engineering papers. For the simplest analytical modeling and preliminary engineering design, the disk is assumed to be homogeneous of constant thickness, solid or annular, and with the only one external centrifugal force applied. Most of the papers consider the Tresca yield criterion and strain hardening material (together with the associated flow rule [4, 5] or deformation theory of plasticity [3]). Another popular approach is to employ the Mises yield criterion and strain hardening material, however, only within the deformation theory of plasticity [6]. Meanwhile, the model of elastic/perfectly-plastic material is commonly excepted in engineering plasticity [7]. In the frame of this model, several important papers have been recently published by Toussi and Farimani [8] (for the Mises yield criterion and deformation theory of plasticity) and Nejad et al. [9] (for the Tresca yield criterion and its associated flow rule). The most recent paper by Lomakin et al. [10] introduces also in analytical treatment the Mises yield criterion combined with its associated flow rule.

In spite of the popularity of the elastic/perfectly-plastic material model, there are serious restrictions on its application when combined with the Tresca yield criterion and its associated flow rule. Particularly, it was shown by Gamer [4] that, for a solid rotating disk free of stresses at the outer contour, this approach leads to a non-admissible discontinuity in radial displacement at the elastic-plastic border. To overcome this inconsistency, Gamer [5] suggested substitution of



elastic/perfectly-plastic material for an elasticplastic one with linear strain hardening property which presumes different forms of the Tresca yield criterion (side and corner regimes) in the plastic zone. Following Gamer's approach, a lot of papers have been published. The modern treatment of a rotating disk problem by this approach is reflected in [11].

Meanwhile, it has been proved by Aleksandrova [12] that it is possible to obtain a continuous solution both for the stresses and radial displacement based on the original elastic/perfectly-plastic material by using the Mises yield criterion and its associated flow rule. A complete stress-strain solution has been then developed for practical engineering applications [13].

It is worth noting that, within the theory of plastic flow, while considering piecewise linear function of plasticity different from the Tresca yield function, stress/displacement fields will be continuous. It is also true, that for an annular disk (not the solid one), the Tresca yield criterion with its associated flow rule leads to continuous stress/displacement fields at the elastic-plastic boundary. To this end, for the piecewise plastic potentials, in the frame work of elastic/perfectly-plastic material model (when more than one plastic regimes are activated), the fulfilment of singular regimes leads to discontinuity in plastic strains on the border of change of the regimes [14].

So, the main objective of the present paper is to develop a consistent analytical model capable to resolve a class of control problems for a rotating solid disk. Due to the importance of the control problems in practical engineering design, recently, some papers have been published to fulfill such a demand. The importance of boundary conditions on optimal shape of annular disk was outlined in [15]. On the other hand, the importance of the material properties on the burst speed of disks was investigated in [16]. To this end, the novelty of the current research is to include two control factors such as rotating speed and external pressure in the consideration of the optimal performance of the disk.

Statement of the problem

The problem of stress/strain state in a thin high-speed rotating disk of constant thickness has been considered in the framework of plane-stress state. Cylindrical coordinate system $\rho\theta z$ (where the axis z passes through the center of the disk $\rho = 0$ and the plane $z = 0$ represents the mid-plane of the disk) is assumed to be suitable for the geometric representation of such a disk. The outer contour of the disk $\rho = b$ is subjected to compressive external pressure p_b . The physical model of the disk is given as isotropic elastic/perfectly-plastic with smooth plasticity functions which are coincident with respective plastic potentials.

Non-dimensional variables

The paper is written in non-dimensional form. All variables of length measure are scaled to the outer radius of the disk b . The values of stress measure are scaled to the yield stress in tension k . However, the notations of both dimensional and respective non-dimensional variables are the same for the sake of reading clarity of the formulas.

Control parameters

The paper deals with the notion of control parameters (which could be external or internal), i.e. parameters upon which the stress/strain state of the disk depends on. It should be mentioned here that this stress/strain state, in turn, is characterized by some set of proper internal parameters different from the internal control parameters.

The external control parameters are related to external actions to which the disk is exposed, such as pressure at the outer contour of the disk and its rotational speed (centrifugal force). The internal control parameters are related to the coefficients in the governing set of equations. Both types of control parameters can vary inside the boundaries established by the model used in the solution of the problem. For a given mathematical model, the changes in internal control

parameters allow to consider certain class of physical bodies. Restrictions on these changes follow from the governing set of equations and general theory of continuum body mechanics. For example, for the Poisson coefficient ν , the limits are $0 \leq \nu \leq 0.5$. The changes in internal control parameters influence the boundaries of allowable external control parameters. Knowing the relationship between the boundaries of allowable external and internal control parameters, it is possible to choose such a material that provides the required working characteristics of the disk performance. It is worth mentioning that there exist two non-dimensional parameters which are equal to unity, namely, $b = 1$ and $k = 1$, so, these parameters do not belong to the control ones. In what follows, all types of control parameters are called simply parameters (if not stated otherwise).

On the choice of yield criterion

The usage of piecewise liner functions of plasticity leads to linear equations from which the stresses are defined. The associated flow rule can be integrated to get so-called associated law of plastic deformations. The challenges related to the choice of piecewise liner functions of plasticity were discussed above.

The usage of smooth functions of plasticity usually does not allow to get analytical solutions, however, in contrast to the piecewise liner functions of plasticity (when several plastic regimes are activated), the smooth functions lead to less complicated algorithm of problem solutions.

It is worth mentioning that, independently of the choice of yield criterion, the solution of the problem without verification of the results (namely, checking all necessary conditions which follow from the given mathematical model) may be wrong.

Determination of stresses in plastic zone

The yield condition is written as

$$\begin{cases} F(\sigma_\rho, \sigma_\theta, \sigma_z) = \alpha k, \\ \sigma_z = 0, \end{cases} \quad (1)$$

where F is the function of plasticity (smooth function), $\sigma_\rho, \sigma_\theta, \sigma_z$ – main stresses, α – parameter equal to the ratio of the given yield stress to the yield stress in tension k . The choice of the yield stress defines the common point of intersection of yield curves $f(\sigma_\rho, \sigma_\theta) = F|_{\sigma_z=0} = \alpha k$. In what follows, without loss of generality, let's assume $\alpha = 1$.

From Eq. (1), in some cases (for example, for linear and quadratic functions of plasticity), it is possible to express the component σ_θ via component σ_ρ .

The governing differential equation of motion is

$$\rho \frac{d\sigma_\rho}{d\rho} + \sigma_\rho - \sigma_\theta + m\rho^2 = 0, \quad (2)$$

where the parameter m characterizes the action of inertia [3].

In the center of the disk, we have

$$\sigma_\rho|_{\rho=0} = \sigma_\theta|_{\rho=0}. \quad (3)$$

If the stresses are known at the elastic-plastic boundary, then the problem in the plastic zone is ecstatically determined [17] and called Cauchy problem.

From the yield condition Eq. (1) and Eq. (3), it follows that the allowable values of parameter p_b are $p_b \in [-k, -\sigma_\rho^{(\min)}]$, where the value $\sigma_\rho^{(\min)}$ is defined from the solution of the system

$$\begin{cases} f(\sigma_\rho, \sigma_\theta) = k, \\ \partial F / \partial \sigma_\theta|_{\sigma_z=0} = 0. \end{cases} \quad (4)$$

On the algorithm of the solution of the problem

The algorithm of the solution of the problem is based on the following considerations. When the plastic zones are about to appear, the external parameters p_b and m should be coherent. This is the reason why, as a first step in the algorithm, the purely elastic problem should be solved from which the points of initiation of plastic regions are determined as well as the relationships between p_b and m .

Then, the limit state of the disk (fully plastic state) is considered. For this state, the problem of determination of coherent values of parameters p_b and m should be analyzed. This approach allows to make rigorous choice of allowable values of parameters p_b and m in the solution of the elastic-plastic disk problem.

Elastic state of the disk

The stresses in the purely elastic disk are well known [1]

$$\sigma_\rho = -\frac{3+\nu}{8}m\rho^2 + A - \frac{B}{\rho^2}, \quad \sigma_\theta = -\frac{1+3\nu}{8}m\rho^2 + A + \frac{B}{\rho^2}, \quad (5)$$

where

$$B = 0, \quad A = -p_b + \frac{3+\nu}{8}mb^2.$$

Let's call the parameters m and p_b allowable for the purely elastic disk, if none of the points of the disk enter into the plastic state. In this case, the images of elastic stress points in a stress space $\sigma_\rho, \sigma_\theta$ are situated inside the plastic curve. On the plane with coordinates $(\sigma_\rho, \sigma_\theta)$, the image of elastic stress points is a segment of line defined by the equation

$$\sigma_\theta = \frac{1+3\nu}{3+\nu}\sigma_\rho + (1-\nu)\left(\frac{mb^2}{4} - \frac{2p_b}{3+\nu}\right). \quad (6)$$

To calculate the elastic stress state, the equivalent stress should be defined first by $\sigma_{eq} = f(\sigma_\rho, \sigma_\theta)$. For the stress state, described by Eqs (5)–(6), the maximum value of the function σ_{eq} can be reached or at the point $\rho = 0$, or at the boundary $\rho = b$, or simultaneously at the point $\rho = 0$ and at the boundary $\rho = b$.

If $p_b = \pm k$, $m = 0$, then the disk is in the limit state – full plasticity with $\sigma_\rho = \sigma_\theta = \mp k$.

The point $\rho = 0$ and boundary $\rho = b$ are in the fully plastic state, if $\sigma_{eq}|_{\rho=0} = k$ and $\sigma_{eq}|_{\rho=b} = k$. The coherent values of parameters m and p_b , when these conditions are fulfilled only at the point $\rho = 0$ and at the boundary $\rho = b$, will be designated as m_k and p_k .

Taking into account Eq. (5), one gets the formula which defines consistent changes in parameter values m and p_b when, at the point $\rho = 0$, the condition $\sigma_{eq} = k$ is satisfied, and, at the all other points, $\sigma_{eq} < k$.

$$\begin{cases} m = m_0 = 8(k + p_b)/(3 + \nu), \\ p_b \in (-k, p_k). \end{cases} \quad (7)$$

Coherent parameter values $m = m_b$ and p_b , when the boundary $\rho = b$ is at the limit state, and, for all other points of the disk, $\sigma_{eq} < k$, are obtained from the solution of the system

$$\begin{cases} \sigma_{eq}|_{\rho=b} = k, \\ p_b \in (p_k, -\sigma_\rho^{(\min)}). \end{cases} \quad (8)$$

In some cases, for values m_b , m_k and p_k , it is possible to get analytical expressions. For example, for the Mises yield criterion, from Eqs (7)–(8), one finds

$$m_b = \frac{2\left(\sqrt{4k^2 - 3p_b^2} + p_b\right)}{(1-\nu)b^2}, \quad m_k = \frac{32(1+\nu)k}{(7\nu^2 + 2\nu + 7)b^2}, \quad p_k = \frac{5 - 3\nu^2 + 14\nu}{7\nu^2 + 2\nu + 7}k. \quad (9)$$

The values m_k and p_k substantially depend on ν such as, from Eq. (9), it follows that

$$\min_{\nu} p_k = \frac{5k}{7}, \quad \min_{\nu} m_k = \frac{32k}{7b^2}, \quad \max_{\nu} p_k = \frac{15k}{13}, \quad \max_{\nu} m_k = \frac{8\sqrt{21}k}{3(\sqrt{21}-7)b^2}.$$

The limit state of elastic-plastic disk

If the disk is in the limit state, then the stresses are found from Eqs (1)–(2) and boundary condition $\sigma_{\rho}|_{\rho=0} = k$. Solving the Cauchy problem, one finds also the value of pressure $p_b = \sigma_{\rho}|_{\rho=b}$ coherent with parameter m .

Since $p_b \in [-k, \sigma_{\rho}^{(\min)}]$, and centrifugal force produces tensile positive radial stress, then the maximum allowable value of parameter $m = m_{\max}$ is defined from the condition $p_b = -\sigma_{\rho}^{(\min)}$. For example, for the Mises yield criterion, $\sigma_{\rho}^{(\min)} = -2k/\sqrt{3}$, so, with a precision 10^{-4} , the value $m_{\max} = 6.276$.

Elastic-plastic state of the disk

As it follows from the above, there are three possible cases for the location of plastic regions, namely, radius location $0 \leq \rho \leq c_1$ ($c_1 \leq b$) or $c_2 \leq \rho \leq b$ ($0 \leq c_2$) or, simultaneously, $0 \leq \rho \leq c_1$ and $c_2 \leq \rho \leq b$ ($c_1 < c_2$).

Inner plastic region $0 \leq \rho \leq c_1$

Inside the plastic zone, the stresses are determined from Eqs (1)–(3). Inside the elastic zone, the stresses are obtained by Eq. (5). If one uses the continuity conditions for stresses at the elastic-plastic boundary $\rho = c_1$ to calculate constants A and B in Eq. (5), then

$$A = \frac{mc_1^2(1+\nu)}{4} + \frac{\sigma_{\theta}^{(1)} - p_{c_1}}{2},$$

$$B = \frac{mc_1^4(\nu-1)}{8} + \frac{c_1^2(\sigma_{\theta}^{(1)} + p_{c_1})}{2},$$

where $\sigma_{\theta}^{(1)}$ and p_{c_1} are the circumferential stress and pressure, respectively, at the unknown elastic-plastic boundary $\rho = c_1$. The boundary condition $\sigma_{\rho}|_{\rho=b} = -p_b$ serves for the determination of the radius of the elastic-plastic boundary.

On the other hand, if the constants A and B are defined from the continuity condition of radial stress at $\rho = c_1$ and boundary condition $\sigma_{\rho}|_{\rho=b} = -p_b$, then

$$A = \frac{mc_1^2((1+3\nu)b^4 + (3+\nu)c_1^4)}{8(b^2 + c_1^2)} + \frac{c_1^2 p_b - b^2 \sigma_{\theta}^{(1)}}{b^2 + c_1^2},$$

$$B = \frac{mc_1^2 b^2((1+3\nu)b^2 + (3+\nu)c_1^2)}{8(b^2 + c_1^2)} - \frac{c_1^2 b^2(p_b + \sigma_{\theta}^{(1)})}{b^2 + c_1^2}.$$

In this case, the radius of the elastic-plastic boundary is derived from the continuity conditions for stresses at the elastic-plastic boundary.

The minimum values of coherent parameters m and p_b are obtained from Eq. (6). The condition $\sigma_{eq}|_{\rho=b} = k$ serves for the determination of coherent values of parameters m and p_b when the disk passes into the limit state ($c_1 = b$).

Outer plastic region $c_2 \leq \rho \leq b$

If the region $c_2 \leq \rho \leq b$ is in the plastic state, then the stresses here is obtained from Eqs (1)–(2) for the condition $\sigma_\rho|_{\rho=b} = -p_b$. The constants A , B and the radius of elastic-plastic boundary are found from the continuity conditions for stresses at the elastic-plastic boundary $\rho = c_2$ and condition $\sigma_\rho|_{\rho=0} = \sigma_\theta|_{\rho=0}$

$$A = p_{c_2} - (3 + \nu)mc_2^2, \quad B = 0, \quad p_{c_2} + (3 + \nu)mc_2^2 - \sigma_\theta^{(2)} = 0,$$

where $\sigma_\theta^{(2)}$ is the circumferential stress at the boundary $\rho = c_2$ from within the plastic zone. The condition $\sigma_{eq}|_{\rho=0} = k$ serves for the determination of coherent values of parameters m and p_b when the disk passes into the limit state ($c_2 = 0$).

Two plastic regions $0 \leq \rho \leq c_1$ and $c_2 \leq \rho \leq b$ ($c_1 \leq c_2$)

For the case considered, the allowable values are $p_b \in [p_k, -\sigma_\rho^{(\min)}]$. The occurrence of two plastic zones is determined by assigning consistent values to $m = m_k$, $p_b = p_k$ defined by Eq. (8). The stresses in plastic zones are obtained the same way as for the individual zones from the solution of the Cauchy problem. The constants A and B , as well as the radii of elastic-plastic boundaries are calculated from the continuity conditions for stresses at these boundaries.

Deformation theory of plasticity

It is well known that due to the deformation theory of plasticity (total strain theory), the tensor of plastic strains is proportional to the deviator of stresses [3]

$$\boldsymbol{\varepsilon}^p = \psi \text{Dev} \boldsymbol{\sigma}. \quad (10)$$

Eq. (10) is, in fact, the consequence of the associated (with the Mises yield criterion) law of plastic deformations

$$\boldsymbol{\varepsilon}^p = \psi \frac{\partial F(\boldsymbol{\sigma})}{\partial \boldsymbol{\sigma}}. \quad (11)$$

For small elastic-plastic strains, one has an additive rule of total, elastic and plastic strains:

$$\varepsilon_\rho = \varepsilon_\rho^e + \varepsilon_\rho^p, \quad \varepsilon_\theta = \varepsilon_\theta^e + \varepsilon_\theta^p.$$

The total strains are related to radial displacement as

$$\varepsilon_\rho = \frac{du}{d\rho}, \quad \varepsilon_\theta = \frac{u}{\rho}. \quad (12)$$

Elastic strains are expressed via stresses conform the Hooke's law

$$E\varepsilon_\rho^e = \sigma_\rho - \nu\sigma_\theta, \quad E\varepsilon_\theta^e = \sigma_\theta - \nu\sigma_\rho, \quad (13)$$

where E is the non-dimensional Young's modulus.

Taking into account the governing equations of the deformation theory of plasticity Eq. (11), and Eqs (12)–(13), one gets the equation for the radial displacement

$$\frac{dEu}{d\rho} = \frac{\partial f / \partial \sigma_\rho}{\partial f / \partial \sigma_\theta} \left(\frac{Eu}{\rho} - \sigma_\theta + \nu\sigma_\rho \right) + \sigma_\rho - \nu\sigma_\theta. \quad (14)$$

From the condition of the symmetry of the displacement field and continuity of displacement at the center of the disk, $u|_{\rho=0} = 0$, it follows that, at the point $\rho = 0$, the ratio $u/\rho = 0/0$ turns

to be indeterminate. Assuming that, at the center of the disk, strains do not attain infinitely large values, one has

$$\lim_{\rho \rightarrow 0} \frac{\partial f / \partial \sigma_\rho}{\partial f / \partial \sigma_\theta} = 1, \quad \lim_{\rho \rightarrow 0} \frac{u}{\rho} = \lim_{\rho \rightarrow 0} \frac{du}{d\rho} = \alpha.$$

By chosen some small values δ , an approximate solution $u = \alpha\rho$ is obtained at the region $0 \leq \rho \leq \delta$ and, for the zone $\delta \leq \rho \leq b$, the Cauchy problem should be solved

$$\begin{cases} \frac{dEu}{d\rho} = \frac{\partial f / \partial \sigma_\rho}{\partial f / \partial \sigma_\theta} \left(\frac{Eu}{\rho} - \sigma_\theta + \nu\sigma_\rho \right) + \sigma_\rho - \nu\sigma_\theta, \\ u|_{\rho=\delta} = \alpha\delta. \end{cases}$$

Aforementioned approach, with an exception of certain details, has been introduced in [18]. Meanwhile, it is possible to get the displacements by another way applying condition $u|_{\rho=b} = u_b$.

It is worth noting that, for the rigid-plastic material model ($E \rightarrow \infty$), the equation for the determination of displacement is homogeneous, so, it will be direct proportionality between parameters α and u_b such as $u_b = \kappa\alpha$. The coefficient κ , which depends on the control parameters, is calculated numerically. For the model of elastic-plastic body, no direct proportionality is observed between α and u_b .

Numerical results

As a numerical example, without loss of generality, let us consider the yield condition proposed in [19, 20]. For the plane-stress state this condition has the form

$$f(\sigma_\rho, \sigma_\theta) = \left(\frac{(\sigma_\rho - \sigma_\theta)^{2n} + \sigma_\theta^{2n} + \sigma_\rho^{2n}}{2} \right)^{\frac{1}{2n}} = k. \quad (15)$$

Parameter n is the internal control parameter. The choice of this condition is related to the fact that one has the Mises yield criterion for $n = 1$ or $n = 2$, and the Tresca yield criterion when $n \rightarrow \infty$.

Figure 1 shows the stress image points in stress space and the shape of the curve defined by Eq. (15) for $n = 4$, $\nu = 0.2$.

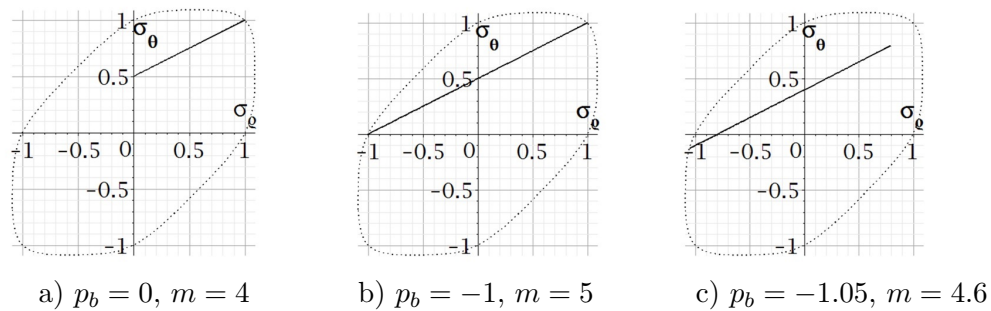


Figure 1. Stress image points in stress space for purely elastic disk.

Figure 2 shows related plots when the plastic zone occupies the central part of the disk.

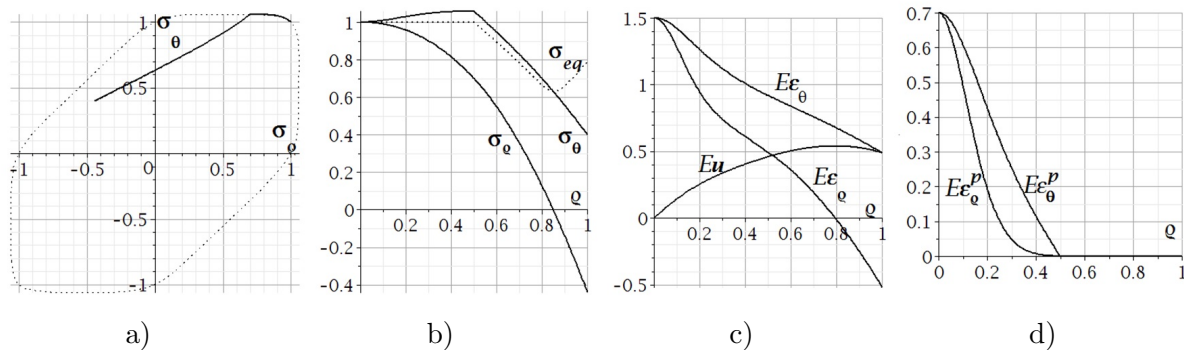


Figure 2. a) stress image points in stress space, b) radial and circumferential stresses, and equivalent stress, c) total strains, d) plastic portions of strains.

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