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OPEN PROBLEMS AND CONJECTURES

Open problems in some competition models

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We present open problems and conjectures for some two-dimensional competition models, namely the logistic competition model and a Ricker-type competition model.

Keywords: Ricker competition model; logistic competition model; stability

1. Ricker competition model

In [2], the authors considered the Ricker competition model given by

$$\begin{cases} x_{n+1} = x_n \, \mathrm{e}^{K-x_n-ay_n} \\ y_{n+1} = y_n \, \mathrm{e}^{L-y_n-bx_n} \end{cases}, \tag{1}$$

where $(x, y) \in \mathbb{R}^2_+$. The parameters $K, L > 0$ are the carrying capacities of species x and y, respectively. Moreover, the competition parameters a and b are in the interval $(0, 1)$. System (1) is based on the popular 1D Ricker map $R(x) = x\,\mathrm{e}^{p-x}$, $p > 0$. The following result is the main stability theorem in [2].

Theorem 1.1. [2] *The coexistence fixed point*

$$(x^*, y^*) = \left(\frac{aL - K}{ab - 1}, \frac{bK - L}{ab - 1}\right)$$

of the Ricker equation (1) *is asymptotically stable if*

$$4(ab - 1) + 2(1 - a)L + 2(1 - b)K \leq (aL - K)(bK - L) < (1 - a)L + (1 - b)K. \tag{2}$$

Equivalently, the coexistence fixed point is asymptotically stable if $(K, L) \in \mathrm{Int}(S_1)$, *with* $a, b \in (0, 1)$, *and* $\mathrm{Int}(S_1)$ *denotes the interior of the region* S_1.

The region S_1, in the parameter space $K - L$, representing (2), is depicted in Figure 1. Now, we have the following conjecture.

Conjecture 1.2. *Under condition (2), the coexistence fixed point* (x^*, y^*) *is globally asymptotically stable in the positive first quadrant, for all* $(K, L) \in \mathrm{Int}(S_1)$.

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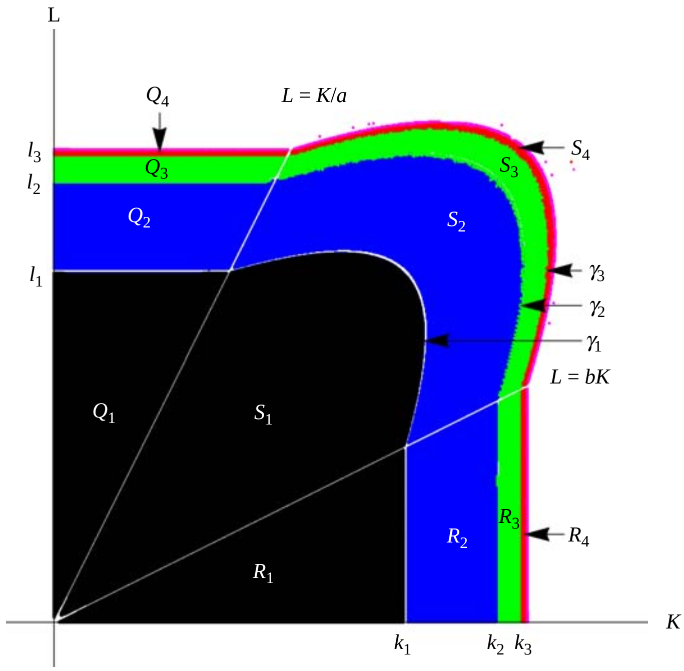

Figure 1. The stability regions and the bifurcation scenario of the Ricker competition equation in the parameter space $K - L$.

Open problem 1.3. Determine the basin of attraction of the periodic cycles of period 2^{n-1}, $n = 2, 3, \ldots$ in regions S_n.

For the exclusion fixed points $(K, 0)$ and $(0, L)$, we have the following result from [2].

Theorem 1.4. [2] *For the Ricker competition equation* (1), *the following statements hold true:*

(1) $(K, 0)$ *is asymptotically stable if* $0 < K \leq 2$ *and* $L < bK$,
(2) $(0, L)$ *is asymptotically stable if* $0 < L \leq 2$ *and* $L > K/a$.

Open problem 1.5.

(1) Determine the stability of the fixed point $(K, 0)$ when $K = 2$ and $L = bK$. In this case, the eigenvalues of the Jacobian of the map of equation (1) are $\lambda_1 = -1$ and $\lambda_2 = 1$.
(2) Determine the stability of the fixed point $(0, L)$ when $L = 2$ and $L = K/a$. In this case, the eigenvalues of the Jacobian of the map of equation (1) are $\lambda_1 = 1$ and $\lambda_2 = -1$.

2. Logistic competition model

In [1], the authors considered the following logistic competition model:

$$\begin{cases} x_{n+1} = \frac{ax_n(1-x_n)}{1+cy_n} \\ y_{n+1} = \frac{by_n(1-y_n)}{1+dx_n} \end{cases}, \tag{3}$$

where $x, y \in [0,1]$, $c, d \in (0,1)$ and $a, b \in (0,4]$. The parameters a and b are called the intrinsic growth rates of species x and y, respectively, and c, d denote the competition of the species.

System (3) has the fixed points $(0,0)$, $((a-1)/a, 0)$, $(0, (b-1)/b)$ and (x^*, y^*), where

$$x^* = \frac{b(a-1) - c(b-1)}{ab - cd}, \quad y^* = \frac{a(b-1) - d(a-1)}{ab - cd}.$$

The Jacobian of system (3) at the fixed point (x^*, y^*) is given by

$$J = \begin{bmatrix} \frac{1-2x^*}{1-x^*} & -\frac{cx^*}{a(1-x^*)} \\ -\frac{\mathrm{d}y^*}{b(1-y^*)} & \frac{1-2y^*}{1-y^*} \end{bmatrix}.$$

We make the assumption that

$$b > 1 + \frac{d(a-1)}{a} \quad \text{and} \quad a > 1 + \frac{c(b-1)}{b}, \tag{4}$$

which insures that the fixed point (x^*, y^*) lies in the positive first quadrant.

Theorem 2.1. [1] *The positive fixed point* (x^*, y^*) *of the Logistic competition equation* (3) *is asymptotically stable if the following conditions hold:*

$$\begin{aligned} &\frac{-c(b-c+bc)d^2 + a^3 b^2(2-b+2d) + a(b-c+bc)d(3b+c+cd)}{ab(-(1+c)d + a(1+d))(-b(1+c) + c(1+d))} \\ &+ \frac{a^2 b\left(2b^2(1+c) + 3c(1+d) - b(3+5d+c(5+4d))\right)}{ab(-(1+c)d + a(1+d))(-b(1+c) + c(1+d))} < 0 \end{aligned} \tag{5}$$

and

$$\begin{aligned} &\frac{-c(b-c+bc)d^2 + a^3 b^2(3-b+3d) - ad\left(-b^2\left(9+14c+5c^2\right) + c^2(1+d)\right)}{ab(-(1+c)d + a(1+d))(-b(1+c) + c(1+d))} \\ &- \frac{adbc(8+4c+4d+3cd)}{ab(-(1+c)d + a(1+d))(-b(1+c) + c(1+d))} \\ &+ \frac{a^2 b\left(3b^2(1+c) + c(9+14d+5d^2) - 3b(3+4d+4c(1+d))\right)}{ab(-(1+c)d + a(1+d))(-b(1+c) + c(1+d))} > 0. \end{aligned} \tag{6}$$

Note that inequality (21) in [1], i.e.

$$\frac{(b(-1+a-c)+c)(a(-1+b-d)+d)(ab-cd)}{ab(-(1+c)d + a(1+d))(-b(1+c) + c(1+d))} < 0, \tag{7}$$

holds true under condition (4). This observation was not noted in [1].

Equivalently, the positive fixed point (x^*, y^*) of equation (3) is asymptotically stable if $(a,b) \in \mathrm{Int}(S_1)$, where S_1 is the region depicted in Figure 2. Note that the curves τ_1 and τ_2

in Figure 2 are defined as

$$\tau_1 = \left\{(a,b) \in \mathbb{R}^2_+ : b = 1 + \frac{d(a-1)}{a}\right\} \text{ and } \tau_2 = \left\{(a,b) \in \mathbb{R}^2_+ : a = 1 + \frac{c(b-1)}{b}\right\}.$$

CONJECTURE 2.2. *The positive fixed point* (x^*, y^*) *of the logistic competition model* (3) *is globally asymptotically stable if* $(a,b) \in \text{Int}(S_1)$.

Open problem 2.3. Determine the basin of attraction of the periodic cycles of period 2^{n-1}, $n = 2, 3, \ldots$ in regions S_n in Figure 2.

For the fixed points $((a-1)/a, 0)$ and $(0, (b-1)/b)$, we have the following result:

THEOREM 2.4. [1] *The following statements hold true:*

(1) *The fixed point* $((a-1)/a, 0)$ *of equation* (3) *is asymptotically stable if* $1 < a \leq 3$ *and* $1 < b < 1 + (d(a-1)/a)$ *and is unstable if* $1 < a < 3$ *and* $b = 1 + (d(a-1)/a)$,
(2) *The fixed point* $(0, (b-1)/b)$ *of equation* (3) *is asymptotically stable if* $1 < b \leq 3$ *and* $1 < a < 1 + (c(b-1)/b)$ *and is unstable if* $1 < b < 3$ *and* $a = 1 + (c(b-1)/b)$.

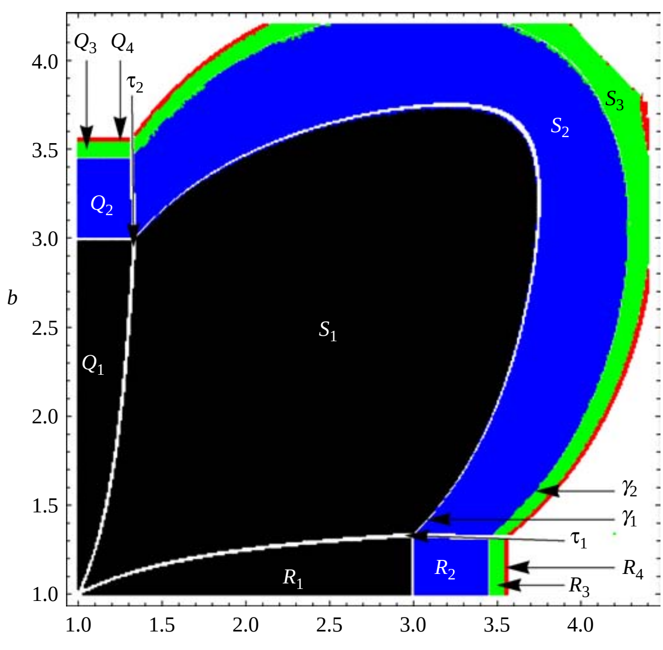

Figure 2. The stability regions and the bifurcation scenario of the competition logistic model in the parameter space $a - b$.

Open problem 2.5.

(1) Determine the stability of the fixed point $((a-1)/a, 0)$ of equation (3) if $a = 3$ and $b = 1 + (d(a-1)/a)$.
(2) Determine the stability of the fixed point $(0, (b-1)/b)$ of equation (3) if $b = 3$ and $a = 1 + (c(b-1)/b)$.

Note

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