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Levi and Harper identities for non-prioritized belief base change

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A B S T R A C T

In this paper, we investigate the relation between shielded base contraction postulates and credibility-limited (CL) base revision postulates. More precisely, we identify (i) the relation between the postulates satisfied by a shielded base contraction operator and the postulates satisfied by the CL base revision operator that is defined from it by means of the consistency-preserving Levi identity and (ii) the relation between the postulates satisfied by a CL base revision operator and the postulates satisfied by the shielded base contraction operator that is defined from it by means of the Harper identity. Furthermore, we show that the consistency-preserving Levi identity and the Harper identity establish a one-to-one correspondence between the twenty classes of shielded base contractions presented in [21] and the twenty classes of credibility-limited base revisions presented in [22].

1. Introduction

Alchourrón, Gärdenfors and Makinson proposed in [1] a belief change framework that is currently commonly designated by the AGM model. In that framework, which has acquired the status of the standard model in the belief change literature, each belief of an agent is represented by a sentence, and the belief state of an agent is represented by a logically closed set of (belief-representing) sentences. In the AGM model, three kinds of change operators for belief sets are considered, namely:

- Expansions, whose output is a belief set that (may be inconsistent) contains all the sentences of the original belief set and the (sentence representing the) new information.
- Contractions, whose output is a belief set that is a subset of the original one, which does not contain the sentence received as input but contains as many of the previous beliefs as possible.
- Revisions, whose output is, whenever possible, a consistent belief set that contains the new belief and as many of the previous beliefs as possible.

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The expansion (usually denoted by $+$) of a belief set by a sentence is a two-step procedure: first, the sentence is added to the belief set, and afterwards, the resulting set is closed by logical consequence. That is, given a belief set $\mathbf{K}$, the outcome of the expansion of $\mathbf{K}$ by α, which is denoted by $\mathbf{K}+\alpha$, is the set $Cn(\mathbf{K}\cup\{\alpha\})$. Contrary to expansions, the operators of contraction and revision are not defined in a unique way but are constrained by a set of postulates. As stated by Meyer [49, p. 18]: "The idea is that these are the rational choices to be made".

The following postulates, which were presented in [1] (following [23,25]), are commonly known as *basic Gärdenfors postulates for contraction* or *basic AGM postulates for contraction*:

($\div$1) $\mathbf{K}\div\alpha = Cn(\mathbf{K}\div\alpha)$, whenever $\mathbf{K}$ is a belief set. (Closure)
($\div$2) $\mathbf{K}\div\alpha \subseteq \mathbf{K}$. (Inclusion)
($\div$3) If $\alpha\notin\mathbf{K}$, then $\mathbf{K}\subseteq\mathbf{K}\div\alpha$. (Vacuity)
($\div$4) If $\nvdash\alpha$, then $\alpha\notin\mathbf{K}\div\alpha$. (Success)
($\div$5) $\mathbf{K}\subseteq(\mathbf{K}\div\alpha)+\alpha$. (Recovery)
($\div$6) If $\vdash\alpha\leftrightarrow\beta$, then $\mathbf{K}\div\alpha=\mathbf{K}\div\beta$. (Extensionality)

The operators that satisfy postulates ($\div$1) to ($\div$6) are known as *basic AGM contractions*.

The following six postulates, which were presented in [26], are commonly known as *basic AGM postulates for revision*[1]:

($\star$1) $\mathbf{K}\star\alpha = Cn(\mathbf{K}\star\alpha)$ (*i.e.*, $\mathbf{K}\star\alpha$ is a belief set). (Closure)
($\star$2) $\alpha\in\mathbf{K}\star\alpha$. (Success)
($\star$3) $\mathbf{K}\star\alpha\subseteq\mathbf{K}+\alpha$. (Inclusion)
($\star$4) If $\neg\alpha\notin\mathbf{K}$, then $\mathbf{K}+\alpha\subseteq\mathbf{K}\star\alpha$. (Vacuity)
($\star$5) If α is consistent, then $\mathbf{K}\star\alpha$ is consistent. (Consistency)
($\star$6) If $\vdash\alpha\leftrightarrow\beta$, then $\mathbf{K}\star\alpha=\mathbf{K}\star\beta$. (Extensionality)

The operators that satisfy postulates ($\star$1) to ($\star$6) are known as *basic AGM revisions*.

The following identity (originally proposed in [45]) allows us to define a revision using a contraction:

Levi identity: $\mathbf{K}\star\alpha = (\mathbf{K}\div\neg\alpha)+\alpha$.

The following equality, which was originally presented in [42], defines a contraction using a revision[2]:

Harper identity: $\mathbf{K}\div\alpha = (\mathbf{K}\star\neg\alpha)\cap\mathbf{K}$.

The AGM model inspired many researchers to propose extensions and generalizations for it (for an overview, see [12] and [13]), as is the case for the following:

(a) Models of belief base change: These are models in which the belief state of an agent is represented by a set of sentences that is not, except as a limiting case, closed under logical consequence. These sets have a fundamental property: they allow us to distinguish between explicit beliefs, which are elements of the belief base and derived beliefs, *i.e.*, elements that are logical consequences of the belief base but that are not explicitly present in the belief base. Belief bases are more suitable than belief sets for representing the belief states of real cognitive (non-omniscient) agents. If we intend to model the way humans (or any other rational agents) reason, the use of belief sets to model epistemic states is inadequate. As Rott pointed out in [53], AGM theory is unrealistic in its assumption that epistemic agents are "ideally competent regarding matters of logic. They should accept all the consequences of the beliefs they hold (that is, their set of beliefs should be logically closed), and they should rigorously see to it that their beliefs are consistent". In the AGM framework, agents have unlimited memory and inference ability, which is an unrealistic feature of human reasoning. Furthermore, as Gärdenfors and Rott pointed out, "when we perform revisions or contractions, it seems that we never do it to the belief set itself (…) but rather on some typically finite base for the belief set" [27]. Additionally, because belief sets are often too large, eventually even infinite, they are not adequate for computational implementations of belief change models. The use of belief bases has been largely studied in the literature ([8,16,30,33,36,52]).
(b) Models of non-prioritized belief change: These are models in which the belief change operators considered do not give priority to the new information received (contrary to what is the case regarding the AGM model, which fulfills the principle of primacy of new information). For example, the output of a non-prioritized revision may not contain the new belief that has motivated that revision. Analogously, the outcome of a non-prioritized contraction may still contain the sentence by which the contraction is made. For an overview, see [13,38].

Regarding the kind of operators mentioned in (b), we highlight the following:

[1] These postulates were previously presented in [1] but with slightly different formulations.

[2] As stated in [23,25], if an operator $\div$ satisfies the contraction postulates ($\div$2) to ($\div$4) and ($\div$6), then the operator $\star$ defined from $\div$ using the Levi identity is a basic AGM revision operator. Conversely, if $\star$ satisfies the revision postulates ($\star$1), ($\star$2), ($\star$4), ($\star$5) and ($\star$6), then the operator $\div$ defined from $\star$ using the Harper identity satisfies the basic AGM contraction postulates.

- The *shielded contraction* operators presented in [11]. These operators are defined using an AGM contraction and a set of sentences, designated by the *set of retractable sentences* and denoted by R. If a sentence α belongs to R, then the outcome of the shielded contraction by it coincides with the outcome of the associated AGM contraction; otherwise, the original belief set is left unchanged.
- The *credibility-limited revision* operators (CL revisions for short) introduced by Hansson et al. [40]. Roughly speaking, a CL revision has the following behavior: If a sentence α is credible, then it is added to the set of beliefs of the agent as a consequence of the revision process; otherwise, no change is made to the belief set. Hence, a CL revision operator is induced by a (standard) revision operator and a set of sentences —which contains the sentences that are considered credible, called the *set of credible sentences* and represented by C. If $\alpha \in C$, then the outcome of the CL revision by it coincides with the outcome of the associated AGM revision; otherwise, the original belief set is left unchanged.[3]

CL revisions and shielded contractions have been extensively studied in the artificial intelligence literature [40,15,5,9,6,57,20,44,50]. In [11], Fermé and Hansson established relationships between shielded contractions and credibility-limited revisions. An operator of revision defined using the Levi identity always satisfies *success*, but CL revisions do not satisfy this postulate in general. For this reason, in [11], the following variant of the Levi identity was proposed for defining an operator (of CL revision) $\odot$ using an operator (of shielded contraction) $\multimap$:

$$\mathbf{K} \odot \alpha = \begin{cases} (\mathbf{K} \multimap \neg\alpha) + \alpha & \text{if } \mathbf{K} \multimap \neg\alpha \nvdash \neg\alpha \\ \mathbf{K} & \text{otherwise} \end{cases} \qquad \textbf{(Consistency-preserving Levi identity)}$$

Currently, an immense amount of information is generated every second. Rational agents, such as governments and companies, must be able to process information to make better decisions. New information is not always reliable. Therefore, a rational agent should not always accept this new information and should have a mechanism to decide which information should be accepted and which should be rejected. At this point, we note that non-prioritized belied change operators, as is the case of shielded base contractions and CL revisions, are useful in the field of artificial intelligence, essentially because they are adequate for modeling the behavior of an agent when confronted with some new information that is inconsistent with his current belief state. We emphasize that by means of this kind of operator, it is possible to obtain more realistic models than those that can be obtained using (only) *prioritized* belief change operators (as is the case of AGM contractions and revisions), since it is naturally expectable that a rational agent will not always be willing to give up any of its present beliefs or to incorporate a new belief (which is inconsistent with its present belief state) even if some external new information compels it to do so. Some concrete settings in which it may be relevant to implement these kinds of operators are, for example, the following: databases (where some integrity constraints are not liable to be retracted); normative systems (in which some norms may not be deleted from the system); and project management (when some restrictions such as budget and schedule cannot be modified when some contingency plan is implemented).

Example 1.1. When working with databases, it is important to detect inconsistent and non-admissible data and prevent them from being incorporated into the system database. To this end, suitable integrity constraints must be defined. These are used by the system's data validation mechanisms to prevent incorrect data from being entered. Integrity constraints are logical conditions that must be satisfied by the input data to be incorporated into the system database. When new input data are received, the database verifies if the integrity constraints are satisfied. If not, the input information is not accepted. In this context, the integrity constraints can be seen as the conditions that define the set of credible sentences. From a different perspective, we can also think of the integrity constraints as "beliefs" that cannot be given up, *i.e.*, as beliefs that do not belong to the set of retractable sentences.

Example 1.2. Sensors allow the collection of data from the surrounding environment, allowing machines to extract useful information to perform appropriate actions. However, the corruption of a sensor can lead to unpleasant consequences since the actions to be taken largely depend on the collected data ([43]). For example, in an industrial environment, a damaged sensor can affect the quality of products, and in a self-driven vehicle, it may result in the loss of human lives. For this reason, it is important to detect faulty sensors as soon as possible. Historical data provided by reliable sensors can be used to define a set of constraints ([54]). Such constraints can also be obtained by means of methods based on machine learning algorithms that are used to detect faulty sensors after data collection ([43]). If the constraints are violated by the information provided by a sensor, then that sensor may be considered defective, and the information provided by it should be treated as unreliable. Otherwise, the information can be considered credible.

Several operators that are contained (simultaneously) in both classes of operators mentioned in items (a) and (b) above have been presented in [15], [21] and [22]. These operators are defined for belief bases and (in general) do not satisfy the postulate of success. In [21], twenty classes of shielded base contraction operators were presented and axiomatically characterized, and in [22], a similar study regarding CL base revision operators was performed. In this paper, we study the interrelation among shielded base contraction postulates and CL base revision postulates in the two following ways:

[3] CL revision can be seen as a modified version of Makinson's Screened revision ([48]).

1. Given a shielded base contraction operator $\sim$ and the CL base revision operator $\circledast_{\sim}$ that is defined from $\sim$ by means (of an adaptation to the belief base context) of the consistency-preserving Harper identity, we identify which postulates of CL revision are *induced* in $\circledast_{\sim}$ by each of the postulates of shielded contraction that are (assumed to be) satisfied by $\sim$.
2. Given a CL base revision operator $\circledast$ and the shielded base contraction operator $\sim_{\circledast}$ that is defined from $\circledast$ using the Levi identity, we identify which postulates of shielded contraction are *induced* in $\sim_{\circledast}$ by each of the postulates of CL revision that are (assumed to be) satisfied by $\circledast$.

Furthermore, we show that there is a one-to-one correspondence between the classes of shielded base contractions presented in [21] and the classes of credibility-limited base revisions presented in [22]. More precisely, we show that:

(i) Each one of the classes of shielded contractions considered in [21] is formed by the shielded contraction operators that can be obtained, by means of the Harper identity, from the CL revision operators that constitute one and only one of the classes of CL revisions introduced in [22].
(ii) Each one of the classes of CL revisions considered in [22] is formed by the CL revision operators that can be obtained, by means (of an adaptation to the belief base context) of the consistency-preserving Levi identity, from the shielded contraction operators that constitute one and only one of the classes of shielded contractions introduced in [21].

The rest of the paper is organized as follows: In Section 2, we introduce the notations and recall the main background concepts that will be needed throughout this article. In Section 3, we present a formal definition of shielded base contraction and recall, from [21], the axiomatic characterizations of several classes of such operators. In Section 4, we present the definition of credibility-limited base revision and recall, from [22], the axiomatic characterizations for several classes of such operators. In Section 5, we study the relations between the properties of a set of retractable sentences and the properties of a set of credible sentences, and we analyze the interrelations among (all) the classes of shielded contractions considered in Section 3, and those of CL revisions considered in Section 4, using the Harper identity and the consistency-preserving Levi identity (adapted to the belief base context). In Section 6, we briefly mention several operators of non-prioritized belief change that have been presented in the literature. In Section 7, we summarize the main contributions of the paper and briefly discuss their relevance. In Appendix A, we provide proofs for all the original results presented.

2. Background

2.1. Formal preliminaries

We will assume a propositional language $\mathcal{L}$ that contains the usual truth functional connectives: $\neg$ (negation), $\wedge$ (conjunction), $\vee$ (disjunction), $\rightarrow$ (implication) and $\leftrightarrow$ (equivalence). We shall make use of a consequence operator Cn that takes sets of sentences to sets of sentences and that satisfies the standard Tarskian properties ([55]), namely: (i) $A \subseteq Cn(A)$ (*inclusion*); (ii) if $A \subseteq B$, then $Cn(A) \subseteq Cn(B)$ (*monotony*) and (iii) $Cn(A) = Cn(Cn(A))$ (*iteration*). Furthermore, we assume that Cn satisfies the following three properties: (iv) if α can be derived from A by classical truth-functional logic, then $\alpha \in Cn(A)$ (*supraclassicality*); (v) $\beta \in Cn(A \cup \{\alpha\})$ if and only if $\alpha \rightarrow \beta \in Cn(A)$ (*deduction*); and (vi) if $\alpha \in Cn(A)$, then $\alpha \in Cn(A')$ for some finite subset A' of A (*compactness*). We will sometimes use $Cn(\alpha)$ for $Cn(\{\alpha\})$, $A \vdash \alpha$ for $\alpha \in Cn(A)$, $\vdash \alpha$ for $\alpha \in Cn(\emptyset)$, $A \nvdash \alpha$ for $\alpha \notin Cn(A)$, $\nvdash \alpha$ for $\alpha \notin Cn(\emptyset)$. The letters $\alpha, \beta, \ldots$ (except for γ and σ) will be used for denoting sentences of $\mathcal{L}$. $A, B, \ldots$ shall denote sets of sentences of $\mathcal{L}$. $\mathbf{K}$ is reserved to represent a set of sentences that is closed under logical consequence (*i.e.*, $\mathbf{K} = Cn(\mathbf{K})$) — such a set is called a *belief set* or *theory*.

2.2. Belief base change postulates

In this section, we present several postulates for belief base change (both for revision and for contraction).

2.2.1. Base contraction postulates

We start by recalling the definition of a contraction operator in terms of postulates presented in [39].

Definition 2.1 *([39]).* An operator $\div$ for a set A is an operator of contraction if and only if $\div$ satisfies the following postulates:

(Success) *([25])* If $\nvdash \alpha$, then $A \div \alpha \nvdash \alpha$.
(Inclusion) *([25])* $A \div \alpha \subseteq A$.

We now recall some well-known base contraction postulates:

(Failure) ([17]) If $\vdash \alpha$, then $A \div \alpha = A$.
(Vacuity) ([24]) If $A \nvdash \alpha$, then $A \subseteq A \div \alpha$.

(Relative Closure) ([35]) $A \cap Cn(A \div \alpha) \subseteq A \div \alpha$.
(Extensionality) ([25]) If $\vdash \alpha \leftrightarrow \beta$, then $A \div \alpha = A \div \beta$.
(Uniformity) ([32]) If it holds for all subsets A' of A that $\alpha \in Cn(A')$ if and only if $\beta \in Cn(A')$ then $A \div \alpha = A \div \beta$.
(Relevance) ([29,32]) If $\beta \in A$ and $\beta \notin A \div \alpha$, then there is a set A' such that $A \div \alpha \subseteq A' \subseteq A$ and $A' \nvdash \alpha$ but $A' \cup \{\beta\} \vdash \alpha$.
(Core-retainment) ([31]) If $\beta \in A$ and $\beta \notin A \div \alpha$ then there is some set A' such that $A' \subseteq A$ and $A' \nvdash \alpha$ but $A' \cup \{\beta\} \vdash \alpha$.
(Disjunctive Elimination) ([14]) If $\beta \in A$ and $\beta \notin A \div \alpha$ then $A \div \alpha \nvdash \alpha \vee \beta$.

The following observation highlights some relations among the postulates presented above.

Observation 2.2 *([39]). Let A be a belief base and $\div$ be an operator on A. Then:*

(a) *If $\div$ satisfies relevance, then it satisfies relative closure and core-retainment.*
(b) *If $\div$ satisfies inclusion and core-retainment, then it satisfies failure and vacuity.*
(c) *If $\div$ satisfies uniformity, then it satisfies extensionality.*

2.2.2. Base revision postulates

The following definition establishes the minimal set of postulates that a revision operator must satisfy.

Definition 2.3 *([22]).* An operator $*$ for a set A is an operator of revision if and only if $*$ satisfies the following postulates:

(Success) *([25])* $\alpha \in A * \alpha$.
(Inclusion) *([25])* $A * \alpha \subseteq A \cup \{\alpha\}$.
(Consistency) *([1])* If $\alpha \nvdash \perp$, then $A * \alpha \nvdash \perp$.

We now recall other belief base revision postulates:

(Vacuity) ([25]) If $A \nvdash \neg\alpha$, then $A \cup \{\alpha\} \subseteq A * \alpha$.
(Weak Relative Closure) ([20]) $A \cap Cn(A \cap A * \alpha) \subseteq A * \alpha$.
(Weak Extensionality) ([20]) If $\vdash \alpha \leftrightarrow \beta$, then $A \cap A * \alpha = A \cap A * \beta$.
(Uniformity) ([34]) If for all subsets $A' \subseteq A$, $A' \cup \{\alpha\} \vdash \perp$ if and only if $A' \cup \{\beta\} \vdash \perp$, then $A \cap (A * \alpha) = A \cap (A * \beta)$.
(Relevance) ([34]) If $\beta \in A$ and $\beta \notin A * \alpha$, then there is some A' such that $A * \alpha \subseteq A' \subseteq A \cup \{\alpha\}$, $A' \nvdash \perp$ but $A' \cup \{\beta\} \vdash \perp$.
(Core-retainment) ([56]) If $\beta \in A$ and $\beta \notin A * \alpha$, then there is some $A' \subseteq A$ such that $A' \nvdash \neg\alpha$ and $A' \cup \{\beta\} \vdash \neg\alpha$.
(Disjunctive Elimination) ([20]) If $\beta \in A$ and $\beta \notin A * \alpha$, then $A * \alpha \nvdash \neg\alpha \vee \beta$.

2.3. Constructive models of base change operators

In this section, we present some explicit definitions of base change functions. As for belief sets, it is possible to define revisions in terms of contractions using the following adaptation of the *Levi identity* to the belief base context:

$$A * \alpha = (A \div \neg\alpha) \cup \{\alpha\}. \qquad \textbf{(Levi identity)}$$

In what follows, we present the explicit definitions of some base contraction and some base revision functions.

2.3.1. Partial meet contraction and revisions

The central concept underlying the definition of partial meet contractions is that of the *remainder set*. The elements of this set are the maximal subsets (of the considered set) that fail to imply a given sentence. Formally:

Definition 2.4 *([2]).* Let A be a belief base and α be a sentence. The set $A \perp \alpha$ (A remainder α) is the set of sets such that $B \in A \perp \alpha$ if and only if:

1. $B \subseteq A$.
2. $B \nvdash \alpha$.
3. There is no set B' such that $B \subset B' \subseteq A$ and $B' \nvdash \alpha$.

The partial meet contractions are obtained by intersecting some elements of the (associated) remainder set. The choice of those elements is performed by means of a *selection function*. A selection function associates to each non-empty remainder set one of its non-empty subsets and to the empty (remainder) set the singleton set $\{A\}$.

Definition 2.5 *([1]).* Let A be a belief base. A selection function for A is a function γ such that for all sentences α:

1. If $A \perp \alpha$ is non-empty, then $\gamma(A \perp \alpha)$ is a non-empty subset of $A \perp \alpha$.

2. If $A\perp\alpha$ is empty, then $\gamma(A\perp\alpha)=\{A\}$.

A partial meet contraction is obtained by intersecting the elements chosen by a selection function.

Definition 2.6 *([1,30]).* The partial meet contraction operator on A based on a selection function γ is the operator $\div_\gamma$ such that for all sentences α:

$$A \div_\gamma \alpha = \cap\gamma(A\perp\alpha).$$

An operator $\div$ for a set A is a partial meet contraction if and only if there is a selection function γ for A such that $A \div \alpha = A \div_\gamma \alpha$ for all sentences α.

Partial meet revisions are the revision functions that can be obtained from partial meet contractions using the Levi Identity.

2.3.2. Kernel contractions and revisions

A *kernel contraction* of a set A by a sentence α consists essentially of the removal of some sentences selected among the sentences of A that contribute effectively to imply α. Formally:

Definition 2.7 *([35]).* Let A be a set in $\mathcal{L}$ and α be a sentence. Then, $A\perp\!\!\!\perp\alpha$ is the set such that $B \in A\perp\!\!\!\perp\alpha$ if and only if:

1. $B \subseteq A$.
2. $B \vdash \alpha$.
3. If $B' \subset B$ then $B' \nvdash \alpha$.

$A\perp\!\!\!\perp\alpha$ is called the kernel set of A with respect to α, and its elements are the α-kernels of A.

To contract a belief α from a set A, one must give up at least one sentence of each α-kernel; otherwise, α would continue being implied by A. The so-called incision functions select the beliefs to be discarded.

Definition 2.8 *([35]).* Let A be a set of sentences. Let $A\perp\!\!\!\perp\alpha$ be the kernel set of A with respect to α. An incision function σ for A is a function such that for all sentences α:

1. $\sigma(A\perp\!\!\!\perp\alpha) \subseteq \bigcup(A\perp\!\!\!\perp\alpha)$.
2. If $\emptyset \neq B \in A\perp\!\!\!\perp\alpha$, then $B \cap \sigma(A\perp\!\!\!\perp\alpha) \neq \emptyset$.

Definition 2.9 *([35]).* Let A be a set of sentences and σ an incision function for A. The kernel contraction on A based on σ is the operator $\div_\sigma$ defined, for any sentence α, by:

$$A \div_\sigma \alpha = A \setminus \sigma(A\perp\!\!\!\perp\alpha).$$

An operator $\div$ for a set A is a kernel contraction if and only if there is an incision function σ for A such that $A \div \alpha = A \div_\sigma \alpha$ for all sentences α.

In the following definition, we recall a more conservative type of kernel contraction, the so-called *smooth kernel contraction*. Sometimes, when contracting a set by a kernel contraction, some beliefs are removed without reason. For example, if $\beta \in A$ and $\beta \in Cn(A \div \alpha)$, then β should also be in $A \div \alpha$ (*i.e.*, $\div$ should satisfy *relative closure*). This holds if the incision function satisfies the condition expressed in the following definition.

Definition 2.10 *([35]).* An incision function σ for a set A is smooth if and only if it holds for all subsets A' of A that if $A' \vdash \beta$ and $\beta \in \sigma(A\perp\!\!\!\perp\alpha)$ then $A' \cap \sigma(A\perp\!\!\!\perp\alpha) \neq \emptyset$.
A kernel contraction is smooth if and only if it is based on a smooth incision function.

Kernel and smooth kernel base revisions are the revision functions that can be obtained from their namesake base contraction operators using the Levi identity.

2.3.3. Basic AGM-generated base contractions and revisions

In what follows, we recall the definitions of other base change operators, the *basic AGM-generated base contractions* and the *basic AGM-generated base revisions*. These are operators defined in terms of basic AGM contractions and basic AGM revisions (for belief sets), respectively.

Definition 2.11 *([14]).* Let A be a belief base. An operator $\div$ for A is a basic AGM-generated base contraction if and only if, for all $\alpha \in \mathcal{L}$:

$$A \div \alpha = (Cn(A) - \alpha) \cap A$$

where $-$ is a basic AGM contraction (*i.e.*, an operator that satisfies the basic AGM postulates for contraction) on $Cn(A)$.[4]

Definition 2.12 *([22]).* Let A be a belief base. An operator $*$ for A is a basic AGM-generated base revision if and only if, for all $\alpha \in \mathcal{L}$:

$$A * \alpha = (Cn(A) \star \alpha) \cap A$$

where $\star$ is a basic AGM revision (*i.e.*, an operator that satisfies the basic AGM postulates for revision) on $Cn(A)$.

3. Shielded base contraction operators

Shielded base contraction operators are defined by means of a base contraction operator and a set of sentences R satisfying certain properties, named the *set of retractable sentences*, which models the set of sentences that the agent is willing to give up (if needed). Informally speaking, a shielded base contraction is a function that receives (just as a *standard* contraction does) a set (of beliefs) and a sentence and returns:

- The received set (unchanged), if the received sentence is not included in R;
- The output produced by the associated base contraction if the received sentence is in R.

Definition 3.1 *([11,15,21]).* Let $\div$ be a contraction operator on a belief base A (*i.e.*, an operator that satisfies success and inclusion). Let R be a set of sentences (the associated set of retractable sentences). Then, $\sim$ is the shielded base contraction induced by $\div$ and R if and only if:

$$A \sim \alpha = \begin{cases} A \div \alpha & \text{if } \alpha \in R \\ A & \text{otherwise} \end{cases}$$

When considering different constraints on the structure of R (the set of retractable sentences) and/or different types of contraction operators, we obtain different kinds of shielded contraction operators.

3.1. The set of retractable sentences

The following properties were proposed for sets of retractable sentences (the first three properties were proposed in [15] and the remaining ones in [21]):

Non-retractability Propagation: If $\alpha \notin R$, then $Cn(\alpha) \cap R = \emptyset$.
Conjunctive Completeness: If $\alpha \wedge \beta \in R$, then $\alpha \in R$ or $\beta \in R$.
Non-retractability of Tautology: $R \cap Cn(\emptyset) = \emptyset$.
Retractability of Logical Equivalents: If $\vdash \alpha \leftrightarrow \beta$, then $\alpha \in R$ if and only if $\beta \in R$.
Uniform Retractability: If it holds for all subsets A' of A that $\alpha \in Cn(A')$ if and only if $\beta \in Cn(A')$, then $\alpha \in R$ if and only if $\beta \in R$.
Non-retractability Upper Bounding: $\mathcal{L} \setminus R \subseteq Cn(A)$.

Non-retractability propagation says that if a sentence α is irretractable, then all its logical consequences are also irretractable. This is a natural property to require from the set R, since the consequences of irretractable beliefs of an agent are also irretractable for that agent. *Conjunctive completeness* states that the conjunction of irretractable sentences is also an irretractable sentence. In fact, to remove a conjunction, we must remove at least one of its conjuncts. *Non-retractability of tautology* states that tautologies are irretractable sentences. *Retractability of logical equivalents* states that two logical equivalent sentences are both retractable or both irretractable. *Uniform retractability* says that if two sentences α and β are implied by exactly the same subsets of A, then they are both retractable or both irretractable. *Non-retractability upper bounding* states that all irretractable sentences are deducible from the set to be contracted.[5]

Additionally, in [21], the following condition, which relates the set of retractable sentences R and the contraction function $\div$ that induce a certain shielded contraction, was proposed:

[4] In [14] these operators were designated by basic related-AGM base contractions.
[5] We note that more rigorously the expression "with respect to A" should be added to the designation of the last two properties presented (namely, *uniform retractability* and *non-retractability upper bounding*), since these relate R and A. However, we will use the shorter designation of these properties since there is no risk of ambiguity whenever these properties are mentioned along this paper.

If $\alpha \notin R$ and $\beta \in R$, then $A \div \beta \vdash \alpha$. **(R - ÷)**

The condition (**R - ÷**) informally illustrates the idea that irretractable sentences should not be removed from the belief base when a shielded contraction (by any sentence) is performed. Then, if a set (of retractable sentences) R and a contraction $\div$ are intended to be used to define a shielded contraction, it should hold that if a sentence is not in R (*i.e.*, is considered irretractable), then it should not be removed when using $\div$ to contract by a sentence included in R (*i.e.*, by a sentence that is considered retractable).[6]

3.2. Postulates for shielded base contractions

The following postulates (that consist of an adaptation to the context of belief bases of the postulates proposed in [11] for shielded contractions on belief sets) were proposed to characterize shielded base contractions.

(Relative success) ([51]) $A \sim \alpha = A$ or $\alpha \notin Cn(A \sim \alpha)$.
(Persistence) ([11]) If $\beta \in Cn(A \sim \beta)$, then $\beta \in Cn(A \sim \alpha)$.
(Success propagation) ([11]) If $A \sim \beta \vdash \beta$ and $\vdash \beta \rightarrow \alpha$, then $A \sim \alpha \vdash \alpha$.
(Conjunctive constancy) ([11]) If $A \sim \alpha = A \sim \beta = A$, then $A \sim (\alpha \wedge \beta) = A$.

Relative success states that when contracting by any given sentence, either that sentence is effectively removed or the belief base is left unchanged. *Persistence* intuitively states that if a belief is removed when contracting a belief base A by some sentence, then it is also removed when A is contracted by that belief itself. *Success propagation* states that if a certain sentence is not removed when trying to contract a belief base by it, then the same thing happens regarding every logical consequence of that sentence. *Conjunctive constancy* states that if the contraction by a given conjunction causes a change, then the same thing happens when contracting by at least one of its conjuncts.

3.3. Representation theorems of some classes of shielded base contractions

We now recall axiomatic characterizations for several classes of shielded contractions. More precisely, we consider the shielded contractions on belief bases induced by partial meet contractions, (smooth) kernel contractions and basic AGM-generated base contractions, and additionally, we take into account different sets of properties regarding the associated set of retractable sentences R.

Observation 3.2 *([21]). Let A be a belief base and $\sim$ be an operator on A. Then:*

$\sim$ is an operator of shielded contraction induced by a	*and a set $R \subseteq \mathcal{L}$ that satisfies*	*iff $\sim$ satisfies relative success, inclusion and*	*Acronym*
partial meet contraction operator $\div$	*uniform retractability*	*uniformity and relevance*	*SPMC*
	uniform retractability and non-retractability propagation	*uniformity, relevance and success propagation*	*SP-SPMC*
	uniform retractability and conjunctive completeness	*uniformity, relevance and conjunctive constancy*	*CC-SPMC*
	uniform retractability, non-retractability propagation and conj. completeness	*uniformity, relevance, success propagation and conjunctive constancy*	*SP+CC-SPMC*
	*condition (**R - ÷**)*	*uniformity, relevance and persistence*	*P-SPMC*
kernel contraction operator $\div$	*uniform retractability*	*uniformity and core-retainment*	*SKC*
	uniform retractability and non-retractability propagation	*uniformity, core-retainment and success propagation*	*SP-SKC*
	uniform retractability and conj. completeness	*uniformity, core-retainment and conj. constancy*	*CC-SKC*
	uniform retractability, non-retractability propagation and conjunctive completeness	*uniformity, core-retainment, success propagation and conj. constancy*	*SP+CC-SKC*
	condition *(**R - ÷**)*	*uniformity, core-retainment and persistence*	*P-SKC*

[6] We note that, rather than a property of R, condition (**R - ÷**) expresses a relation between the set R and a (standard) contraction function on a belief base A.

∼ is an operator of shielded contraction induced by a	and a set $R \subseteq \mathcal{L}$ that satisfies	iff ∼ satisfies relative success, inclusion and	Acronym
smooth kernel contraction operator $\div$	uniform retractability	uniformity, core-retainment and relative closure	SSKC
	uniform retractability and non-retractability propagation	uniformity, core-retainment relative closure and success propagation	SP-SSKC
	uniform retractability and conj. completeness	uniformity, core-retainment, relative closure and conj. constancy	CC-SSKC
	uniform retractability, non-retractability propagation and conjunctive completeness	uniformity, core-retainment, relative closure, success propagation and conj. constancy	SP+CC-SSKC
	condition (**R** - $\div$)	uniformity, core-retainment relative closure and persistence	P-SSKC
basic AGM-generated base contraction operator $\div$	retractability of logical equivalents	vacuity, extensionality and disjunctive elimination	SbAGMC
	non-retractability propagation	vacuity, extensionality, disj. elimination and success propagation	SP-SbAGMC
	retr. of logical equivalents and conj. completeness	vacuity, extensionality, disj. elimination and conj. constancy	CC-SbAGMC
	non-retractability propagation and conjunctive completeness	vacuity, extens., disj. elimination, success prop. and conj. constancy	SP+CC-SbAGMC
	condition (**R** - $\div$)	vacuity, extens., disj. elimination and persistence	P-SbAGMC

We finish this section by presenting examples of shielded contraction operators that belong to some of the classes mentioned in the above theorem but do not belong to the others.

Example 3.3. Let $A = \{p, p \vee q, p \to q\}$ and Cn be purely truth-functional. It holds that $A \perp\!\!\!\perp q = \{\{p, p \to q\}, \{p \vee q, p \to q\}\}$. Let $R = \mathcal{L} \backslash Cn(p \vee q)$ and $\div$ be a kernel contraction based on an incision function σ such that for all sentences $\alpha \in R$, it holds that $\sigma(\alpha) \subseteq R$,[7] and, in particular, $\sigma(q) = \{p, p \to q\}$. Let $\sim$ be the shielded contraction induced by $\div$ and R. Since the set R satisfies *conjunctive completeness*, *non-retractability propagation* and *uniform retractability*, and $\div$ and R satisfy condition *(**R** - $\div$)* (see Lemma 1 in Appendix A), $\sim$ is an SP+CC-SKC and a P-SKC. On the other hand, $A \sim q = A \div q = \{p \vee q\}$, and from $p \in A \setminus A \div q$, it follows that $\sim$ does not satisfy *relevance* or *disjunctive elimination*. Thus, $\sim$ is not an SPMC or an SbAGMC.

4. Credibility-limited base revision operators

The basic idea underlying the definition of credibility-limited (CL) revision is a two-step procedure. The first step consists of the identification of credible sentences, *i.e.*, the sentences that an agent is willing to incorporate when performing a revision. The second step consists of the following behavior:

- leave the revised set of beliefs unchanged when revising by a non-credible sentence; and
- work as a "standard" revision when revising by a credible sentence.

The following definition formalizes this concept:

Definition 4.1 *([15,22,40])*. Let $*$ be a revision operator on a belief base A (*i.e.*, an operator that satisfies success, inclusion and consistency). Let C be a set of sentences (the associated set of credible sentences). Then, $\circledast$ is a CL base revision induced by $*$ and C if and only if:

$$A \circledast \alpha = \begin{cases} A * \alpha & \text{if } \alpha \in C \\ A & \text{otherwise} \end{cases} .$$

When considering different restrictions on the structure of C (the set of credible sentences) and/or different kinds of revision operators, we obtain different types of CL base revision operators.

[7] Note that if $\alpha \in R$, then $\alpha \notin Cn(p \vee q)$. Then, any α-kernel of A that contains $p \vee q$ must contain at least one sentence of $A \setminus Cn(p \vee q)$. For this reason it is possible to define an incision function σ such that for all $\alpha \in R$, $\sigma(\alpha) \cap Cn(p \vee q) = \emptyset$.

4.1. The set of credible sentences

In the CL model, the set of sentences that an agent is willing to accept when a revision is performed is called the *set of credible sentences*. This set will be denoted by C.[8] In [40], some desirable properties for set C were presented.

Credibility of logical equivalents: If $\vdash \alpha \leftrightarrow \beta$, then $\alpha \in C$ if and only if $\beta \in C$.[9]
Single sentence closure: If $\alpha \in C$, then $Cn(\alpha) \subseteq C$.
Disjunctive completeness: If $\alpha \vee \beta \in C$, then either $\alpha \in C$ or $\beta \in C$.
Element consistency: If $\alpha \in C$, then $\alpha \nvdash \bot$.
Expansive credibility: If $A \nvdash \alpha$, then $\neg\alpha \in C$.

In [22], the following properties for C (the set of credible sentences) were proposed.

Credibility lower bounding: If A is consistent, then $Cn(A) \subseteq C$.
Uniform credibility: If it holds for all subsets A' of A that $A' \cup \{\alpha\} \vdash \bot$ if and only if $A' \cup \{\beta\} \vdash \bot$, then $\alpha \in C$ if and only if $\beta \in C$.

Credibility of logical equivalents imposes that equivalent sentences have the same credibility status. *Single sentence closure* states that if we assign the status of "credible" to a sentence α, then the same should happen to its logical consequences. *Disjunctive completeness* states that if two sentences are both not credible, then their disjunction is also not credible. *Element consistency* states that contradictions are not credible. *Expansive credibility* informally states that if a belief is consistent with the agent's belief base, then it is credible. *Credibility lower bounding* states that all the logical consequences of a consistent belief base are credible. *Uniform credibility* states that if two sentences have the "same behavior" regarding a belief base A, *i.e.*, are consistent with exactly the same subsets of A, it is expected that both have the same credibility status.[10]

Additionally, in [22], the following condition, which relates the set of credible sentences C and the revision function $*$ that induce a given credibility-limited operator, was proposed.

$$\text{If } \alpha \notin C \text{ and } \beta \in C, \text{ then } A \cap A * \beta \vdash \neg\alpha. \qquad (\mathbf{C}\text{ - }*)$$

Condition ($\mathbf{C}$ - $*$) states that if a sentence α is not credible, then any possible outcome of revising a set A by a credible sentence contains a subset of A that implies $\neg\alpha$.[11]

4.2. Postulates for credibility-limited base revisions

In this subsection, we recall some postulates for (credibility-limited) base revision and some relations among these postulates. The following postulates are an adaptation to the belief bases context of the postulates for credibility-limited revisions on belief sets presented in [40].

(Relative Success) $\alpha \in A \circledast \alpha$ or $A \circledast \alpha = A$.
(Strict Improvement) If $\alpha \in A \circledast \alpha$ and $\vdash \alpha \rightarrow \beta$, then $\beta \in A \circledast \beta$.
(Disjunctive Distribution) If $\alpha \vee \beta \in A \circledast (\alpha \vee \beta)$, then $\alpha \in A \circledast \alpha$ or $\beta \in A \circledast \beta$.
(Consistency Preservation) If $A \nvdash \bot$, then $A \circledast \alpha \nvdash \bot$.

The following postulate was proposed in [20,22].

(Persistence) If $\neg\beta \in Cn(A \cap A \circledast \beta)$, then $\neg\beta \in Cn(A \cap A \circledast \alpha)$.

Relative success states that when revising by any sentence, either that sentence is contained in the resulting belief base or the original belief base is left unchanged. *Strict improvement* states that if a certain sentence is incorporated when revising a belief base by it, then the same thing happens regarding every logical consequence of that sentence. *Disjunctive distribution* states that if a disjunction belongs to the revision of a belief base by it, then the same thing happens regarding at least one of its disjuncts. *Consistency Preservation* states that if a belief base is consistent, then so is the outcome of any revision of that belief base. *Persistence* states that if the formulae of A that are kept when revising it by a sentence β imply $\neg\beta$, then $\neg\beta$ is implied by the formulae of A that remain when revising it by any formula.

[8] Having in mind Example 1.2, the information provided by a sensor that satisfies the constraints provided by the historical data (which determine that the sensor is not defective), can be seen, in this context, as being "Credible Sentences".
[9] This property was named *closure under logical equivalence* in [40].
[10] We note that more rigorously the expression "with respect to A" should be added to the designation of the last three properties. This will be omitted since there is no risk of ambiguity whenever these properties are mentioned along this paper.
[11] We note that, rather than a property of the set C, condition ($\mathbf{C}$ - $*$) is an expression which relates the set C and a (standard) base revision on a belief base A.

Although *persistence* is a reasonable property (of shielded contraction and credibility-limited revision), there are some contexts in which this postulate is not satisfied, as is the case in the following situation.[12]

Example 4.2 *([22]).* Suppose I believe strongly in $\neg\beta$ and that at a certain moment I get the information that β is the case. I am not willing to stop believing in $\neg\beta$ in order to incorporate β. Suppose now that I get stronger information that proves that β is the case (for example, $\alpha \wedge (\alpha \rightarrow \beta)$). Therefore, although I revise by a belief other than β, I end up losing my belief in $\neg\beta$. For example, suppose John made a retreat away from the rest of the world between December 2019 and May 2020 and did not have access to any information during that period. Suppose now that when he returns to civilization, in June 2020, we tell him that the streets of some of the main cities of the world have been completely empty for several weeks in March-April 2020 (β). John will not believe it. Then, we explain that this happened because a virus appeared in December 2019 (α), and to avoid contamination, a large part of the population had to be confined at home ($\alpha \rightarrow \beta$). Then, John may recall hearing in the news, before going to the retreat, that a highly contagious virus had been discovered. At that moment, John will start to believe that the streets of some of the main cities in the world have been completely empty for some time during his retreat (β).

The following observation states that weak extensionality follows from uniformity.

Observation 4.3 *([20]). Let A be a belief base and $\circledast$ be an operator on A. If $\circledast$ satisfies uniformity, then $\circledast$ satisfies weak extensionality.*

4.3. Representation theorems for some classes of credibility-limited base revisions

We now recall axiomatic characterizations for several classes of CL base revisions. More precisely, we consider the CL revisions on belief bases induced by partial meet revisions, by (smooth) kernel revisions and by basic AGM-generated base revisions, taking into account different sets of properties regarding the associated set of credible sentences.

Observation 4.4 *([22]). Let A be a consistent belief base and $\circledast$ be an operator on A. Then:*

$\circledast$ is an operator of CL base revision induced by a	*and a set $C \subseteq \mathcal{L}$ that satisfies element consistency, expansive credibility and*	*iff $\circledast$ satisfies relative success, consistency preservation, inclusion, vacuity and*	*Acronym*
partial meet revision operator $*$	*uniform credibility*	*uniformity and relevance*	*CLPMR*
	uniform credibility and single sentence closure	*uniformity, relevance and strict improvement*	*SI-CLPMR*
	uniform credibility and disjunctive completeness	*uniformity, relevance and disjunctive distribution*	*DD-CLPMR*
	uniform credibility, single sentence closure and disj. completeness	*uniformity, relevance, strict improvement and disj. distribution*	*SI+DD-CLPMR*
	condition (C - $*$)	*uniformity, relevance and persistence*	*P-CLPMR*
kernel revision operator $*$	*uniform credibility*	*uniformity and core-retainment*	*CLKR*
	uniform credibility and single sentence closure	*uniformity, core-retainment and strict improvement*	*SI-CLKR*
	uniform credibility and disjunctive completeness	*uniformity, core-retainment and disjunctive distribution*	*DD-CLKR*
	uniform credibility, single sentence closure and disj. completeness	*uniformity, core-retainment, strict improvement and disjunctive distribution*	*SI+DD-CLKR*
	condition (C - $*$)	*uniformity, core-retainment and persistence*	*P-CLKR*
smooth kernel revision operator $*$	*uniform credibility*	*uniformity, core-retainment and weak relative closure*	*CLSKR*
	uniform credibility and single sentence closure	*uniformity, core-retainment, weak relative closure and strict improvement*	*SI-CLSKR*
	uniform credibility and disjunctive completeness	*uniformity, core-retainment, weak relative closure and disj. distribution*	*DD-CLSKR*
	uniform credibility, single sentence closure and disj. completeness	*uniformity, core-retainment, weak relative closure, strict improvement and disj. distribution*	*SI+DD-CLSKR*
	condition (C - $*$)	*uniformity, core-retainment, weak relative closure and persistence*	*P-CLSKR*

(*continued on next page*)

[12] For this reason, we consider several classes of shielded base contractions and of credibility-limited base revision, some of which satisfy *persistence* and others that do not.

⊛ is an operator of CL base revision induced by a	and a set $C \subseteq \mathcal{L}$ that satisfies element consistency, expansive credibility and	iff ⊛ satisfies relative success, consistency preservation, inclusion, vacuity and	Acronym
Basic AGM-Generated base revision operator $*$	credibility of logical equivalents	weak extensionality and disjunctive elimination	CLbAGMR
	single sentence closure	weak extensionality, disj. elimination and strict improvement	SI-CLbAGMR
	cred. of logical equivalents and disj. completeness	weak extensionality, disj. elimination and disjunctive distribution	DD-CLbAGMR
	single sentence closure and disj. completeness	weak extensionality, disjunctive elimination, strict improvement and disj. distribution	SI+DD-CLbAGMR
	condition (**C - $*$**)	weak extensionality, disj. elimination and persistence	P-CLbAGMR

Next, we present an example of a credibility-limited revision operator that belongs to some of the classes mentioned in the theorem above but does not belong to all of them.

Example 4.5. Let $A = \{p, q\}$, Cn be purely truth-functional and $*$ be a partial meet revision on A such that $A * (\neg p \vee \neg q) = \{p, \neg p \vee \neg q\}$ and $A * (\neg p \wedge \neg q) = \{\neg p \wedge \neg q\}$. Let ⊛ be the operator of credibility-limited base revision induced by $*$ and the set $C = \{\alpha \in \mathcal{L} : \neg\alpha \notin Cn(p) \cup Cn(q)\}$. This set C satisfies *element consistency, expansive credibility, uniform credibility* and *single sentence closure* (see Lemma 2 in Appendix A). Therefore, ⊛ is a CLPMR and a SI-CLPMR. On the other hand, it holds that $\neg p \notin C$, $\neg q \notin C$ and $\neg p \vee \neg q \in C$. Hence, $A \circledast \neg p = A \circledast \neg q = A$ but $A \circledast (\neg p \vee \neg q) = A * (\neg p \vee \neg q) = \{p, \neg p \vee \neg q\}$. Thus, ⊛ does not satisfy *disjunctive distribution*. Therefore, ⊛ is not a DD-CLPMR or a SI+DD-CLPMR.

5. Relations between shielded base contractions and credibility-limited base revisions

In this section, we establish the relations between the properties of a set of retractable sentences R and of a set of credible sentences C. We also establish the relation between different kinds of operators of shielded contractions and credibility-limited revisions using the consistency-preserving Levi identity and the Harper identity (adapted to the belief base context).

5.1. Relations between sets of credible and retractable sentences

In this subsection, we study the relation between credible and retractable sentences of an agent. If we want to ensure that our credibility-limited revision operators satisfy *consistency preservation*, then we must ensure that a sentence is credible only if its negation can be removed during the revision process; otherwise, the outcome of this revision will be inconsistent. Hence, we can relate the sets R and C by the following condition: if $\alpha \in C$, then $\neg\alpha \in R$. On the other hand, if $\neg\alpha \in R$ then, when revising by α, there is no reason to keep $\neg\alpha$ (since the agent is predisposed to remove it), and so there is no reason not to accept α. Thus, one should also expect that if $\neg\alpha \in R$, then $\alpha \in C$. Therefore, the following condition, originally proposed in [15], seems natural:

$\alpha \in C$ if and only if $\neg\alpha \in R$. (**C-R**)

The following condition can be seen as the dual of the previous one:

$\alpha \in R$ if and only if $\neg\alpha \in C$. (**R-C**)

The following observation illustrates that conditions (**C-R**) and (**R-C**) are equivalent provided that R and C are closed under double negation.

Observation 5.1. *Let R and C be subsets of $\mathcal{L}$. R is closed under double negation, and condition (**C-R**) holds if and only if C is closed under double negation, and condition (**R-C**) holds.*

Having in mind conditions (**C-R**) and (**R-C**), relating credible and retractable sentences, the following observations establish the relation between properties of sets of retractable sentences and of credible sentences.

Observation 5.2. *Let A be a belief base and R and C be sets of sentences that are closed under double negation and satisfy conditions (**C-R**) and (**R-C**).*[13] *It holds that:*

[13] Note that, according to Observation 5.1, conditions (**C-R**) and (**R-C**) are equivalent when C and R are closed under double negation.

(a)

R satisfies	*if and only if C satisfies*
retractability of logical equivalents	*credibility of logical equivalents*
non-retractability of tautology	*element consistency*
non-retractability propagation	*single sentence closure*
uniform retractability with respect to A	*uniform credibility with respect to A*
non-retractability upper bounding with respect to A	*expansive credibility with respect to A*

(b) *If R satisfies* retractability of logical equivalents *and C satisfies* credibility of logical equivalents*, then*[14]*:*

R satisfies	*if and only if C satisfies*
conjunctive completeness	*disjunctive completeness*

The following observation relates conditions (**R - $\div$**) and (**C - $*$**), when C and R are related through condition (**C-R**) and the revision operator $*$ is defined from $\div$ using the Levi identity.

Observation 5.3. *Let A be a belief base and R and C be sets of sentences that satisfy condition (**C-R**). Let $*$ be a revision operator defined from the contraction operator $\div$ on A using the Levi identity. If R and $\div$ satisfy condition (**R - $\div$**), then C and $*$ satisfy condition (**C - $*$**).*

The following observation relates conditions (**C - $*$**) and (**R - $\div$**), whenever the contraction operator $\div$ is defined from $*$ using the Harper identity and C and R are related through condition (**R-C**):

Observation 5.4. *Let A be a belief base and R and C be sets of sentences that satisfy condition (**R-C**). Let $\div$ be a contraction operator defined from the revision operator $*$ on A using the Harper identity. If C and $*$ satisfy condition (**C - $*$**), then R and $\div$ satisfy condition (**R - $\div$**).*

5.2. Generalized Levi and Harper identities

In this subsection, we establish several results that relate credibility-limited revisions and shielded contractions through the consistency-preserving Levi identity (adapted to the belief base context)

$$A \circledast \alpha = \begin{cases} (A \sim \neg\alpha) \cup \{\alpha\} & \text{if } A \sim \neg\alpha \nvdash \neg\alpha \\ A & \text{otherwise} \end{cases} \qquad \textbf{(C-P Levi identity)}$$

and the Harper identity

$$A \sim \alpha = (A \circledast \neg\alpha) \cap A. \qquad \textbf{(Harper identity)}$$

The following theorem illustrates that if a contraction operator $\div$ is defined from a revision operator $*$ using the Harper identity, then the shielded contraction induced by $\div$ and R can be obtained, using the Harper identity, from the credibility-limited revision operator induced by $*$ and C, provided that the sets R and C are related by through condition (**R-C**).

Theorem 5.5. *Let A be a belief base and $*$ be a revision operator on A. Let $\div$ be the contraction operator on A defined from $*$ using the Harper identity. Let $C \subseteq \mathcal{L}$ and $\circledast$ be the credibility-limited revision induced by $*$ and C. Let $R \subseteq \mathcal{L}$ be the set defined from C using condition (**R-C**). Let $\sim$ be the shielded base contraction on A induced by $\div$ and R. Then, $\sim$ can be defined from $\circledast$ using the Harper identity.*

In the following theorem, a result that can be seen as the dual of the previous one is presented. The second item of this theorem states that if a revision operator $*$ is defined from a contraction operator $\div$ using the Levi identity, then the credibility-limited revision induced by $*$ and C can be obtained using the consistency-preserving Levi identity from the shielded contraction operator induced by $\div$ and R, provided that the sets R and C are related by the condition (**C-R**) and R satisfies *non-retractability of tautology* and *non-retractability upper bounding*.

Theorem 5.6. *Let A be a belief base and $\div$ be a contraction operator on A. Let $*$ be the revision operator on A defined from $\div$ using the Levi identity. Let $R \subseteq \mathcal{L}$ and $\sim$ be the shielded base contraction induced by $\div$ and R. Let $C \subseteq \mathcal{L}$ be the set defined from R using condition (**C-R**). Let $\circledast$ be the credibility-limited revision induced by $*$ and C. Then:*

[14] Note that, according to (a), under the current assumptions, R satisfies *retractability of logical equivalents* if and only if C satisfies *credibility of logical equivalents*.

(a)

$$A \circledast \alpha = \begin{cases} (A \sim \neg\alpha) \cup \{\alpha\} & \textit{if } \alpha \in C \\ A & \textit{otherwise} \end{cases}$$

(b) *If R satisfies non-retractability of tautology and non-retractability upper bounding, then $\circledast$ can be defined from $\sim$ using the consistency-preserving Levi identity.*

The following two theorems illustrate some relations between the postulates of shielded contraction and credibility-limited revision whenever one of these operators is obtained from the other using the Harper or the consistency-preserving Levi identities.

Theorem 5.7. *Let A be a consistent belief base and $\sim$ be a shielded base contraction on A. Let $\circledast$ be defined from $\sim$ via the consistency-preserving Levi identity. Then:*

If $\sim$ satisfies	*then $\circledast$ satisfies*
–	*relative success and consistency preservation*
inclusion	*inclusion*
inclusion and persistence	*disjunctive distribution and persistence*
inclusion, vacuity and uniformity	*uniformity*
relevance	*relevance*
core-retainment	*core-retainment*
conjunctive constancy, relative success and extensionality	*disjunctive distribution*
inclusion and success propagation	*strict improvement*
inclusion and vacuity	*vacuity*
disjunctive elimination	*disjunctive elimination*
extensionality, inclusion and vacuity	*weak extensionality*
inclusion, vacuity and relative closure	*weak relative closure*

Theorem 5.8. *Let A be a consistent belief base and $\circledast$ be a credibility-limited base revision on A. Let $\sim$ be defined from $\circledast$ via the Harper identity. Then:*

If $\circledast$ satisfies	*then $\sim$ satisfies*
–	*inclusion*
relative success and consistency preservation	*relative success*
persistence	*persistence*
relative success and relevance	*relevance*
core-retainment	*core-retainment*
uniformity	*uniformity*
vacuity	*vacuity*
vacuity, relative success, consistency preservation, disjunctive distribution and weak extensionality	*conjunctive constancy*
disjunctive elimination	*disjunctive elimination*
weak extensionality	*extensionality*
consistency preservation, strict improvement and relative success	*success propagation*
weak relative closure	*relative closure*

The following theorem clarifies that each element of one of the classes of shielded contraction considered in Section 3 gives rise, using the consistency-preserving Levi identity, to an element of one of the classes of credibility-limited base revision operators mentioned in the previous section.

Theorem 5.9. *Let A be a consistent belief base and $\sim$ be a shielded base contraction operator on A. Let $\circledast$ be defined from $\sim$ via the consistency-preserving Levi identity. Then:*

(a)

If $\sim$ *is a*	*then* $\circledast$ *is a*
SPMC	*CLPMR*
SP-SPMC	*SI-CLPMR*
CC-SPMC	*DD-CLPMR*
SP+CC-SPMC	*SI+DD-CLPMR*
P-SPMC	*P-CLPMR*

(b)

If $\sim$ *is a*	*then* $\circledast$ *is a*
SKC	*CLKR*
SP-SKC	*SI-CLKR*
CC-SKC	*DD-CLKR*
SP+CC-SKC	*SI+DD-CLKR*
P-SKC	*P-CLKR*

(c)

If $\sim$ *is a*	*then* $\circledast$ *is a*
SSKC	*CLSKR*
SP-SSKC	*SI-CLSKR*
CC-SSKC	*DD-CLSKR*
SP+CC-SSKC	*SI+DD-CLSKR*
P-SSKC	*P-CLSKR*

(d)

If $\sim$ *is a*	*then* $\circledast$ *is a*
SbAGMC	*CLbAGMR*
SP-SbAGMC	*SI-CLbAGMR*
CC-SbAGMC	*DD-CLbAGMR*
SP+CC-SbAGMC	*SI+DD-CLbAGMR*
P-SbAGMC	*P-CLbAGMR*

The following theorem illustrates that each element of one of the classes of credibility-limited base revision operators considered in Section 4 gives rise, using the Harper identity, to an element of the classes of shielded contractions considered in Section 3.

Theorem 5.10. *Let A be a consistent belief base and* $\circledast$ *be a credibility-limited base revision operator on A. Let* $\sim$ *be defined from* $\circledast$ *via the Harper identity. Then:*

(a)

If $\circledast$ *is a*	*then* $\sim$ *is a*
CLPMR	*SPMC*
SI-CLPMR	*SP-SPMC*
DD-CLPMR	*CC-SPMC*
SI+DD-CLPMR	*SP+CC-SPMC*
P-CLPMR	*P-SPMC*

(b)

If $\circledast$ *is a*	*then* $\sim$ *is a*
CLKR	*SKC*
SI-CLKR	*SP-SKC*
DD-CLKR	*CC-SKC*
SI+DD-CLKR	*SP+CC-SKC*
P-CLKR	*P-SKC*

(c)

If $\circledast$ *is a*	*then* $\sim$ *is a*
CLSKR	*SSKC*
SI-CLSKR	*SP-SSKC*
DD-CLSKR	*CC-SSKC*
SI+DD-CLSKR	*SP+CC-SSKC*
P-CLSKR	*P-SSKC*

(d)

If $\circledast$ *is a*	*then* $\sim$ *is a*
CLbAGMR	*SbAGMC*
SI-CLbAGMR	*SP-SbAGMC*
DD-CLbAGMR	*CC-SbAGMC*
SI+DD-CLbAGMR	*SP+CC-SbAGMC*
P-CLbAGMR	*P-SbAGMC*

Next, we will show that the operators of non-prioritized base contraction and revision considered in this paper are interdefinable through the Harper and the consistency-preserving Levi identities. The following definition introduces functions that take us from contractions to revisions and vice-versa.

Definition 5.11 *([47,11]).* For every operator $-\!\!\!\!\ominus$, $\mathbb{R}(-\!\!\!\!\ominus)$ is the operator generated from $-\!\!\!\!\ominus$ through the consistency-preserving Levi identity. Furthermore, for every operator $\odot$, $\mathbb{C}(\odot)$ is the operator generated from $\odot$ using the Harper identity.

The following theorems illustrate that operators of shielded base contraction and CL base revision are interdefinable through the Harper and the consistency-preserving Levi identities.

Theorem 5.12. *Let A be a consistent belief base and* $\sim$ *be an operator for A that satisfies the (shielded contraction) postulates of inclusion, vacuity, extensionality and relative success. Then,* $\mathbb{C}(\mathbb{R}(\sim)) = \sim$.

Theorem 5.13. *Let A be a consistent belief base and* $\circledast$ *be an operator for A that satisfies the (credibility-limited revision) postulates of relative success, consistency preservation, inclusion, vacuity and weak extensionality. Then,* $\mathbb{R}(\mathbb{C}(\circledast)) = \circledast$.

6. Related works

In this section, we will mention other approaches related to the present paper. We will divide the related works into three groups: (a) non-prioritized contraction, (b) non-prioritized revision, and (c) status of the belief regarding its credibility/retractability.

(a) Shielded contraction was originally defined and axiomatically characterized in [11]. In this paper, the relation between Shielded and Credibility-Limited operators was established. In [15], the basic construction of shielded contraction was extended for belief bases. Later, in [21], twenty classes of shielded base contraction operators were characterized (based on partial meet base contraction, kernel and smooth kernel base contraction and basic AGM-generated base contraction operators). To the best of our knowledge, no other non-prioritized contraction operators have been proposed in the literature.

(b) The model of credibility-limited revision operators has been largely studied in the literature. In the original paper [40], it was axiomatically characterized, and it was also developed in terms of partial meet contraction, epistemic entrenchment and possible world models. In [15], the basic credibility-limited revision construction was extended for belief bases. Later, in [22], twenty classes of credibility-limited base revision operators were characterized (based on partial meet base revision, kernel and smooth kernel base revision and basic AGM-generated base revision operators). In [5], iterated credibility-limited revision operators were studied.

In the classification for non-prioritized revision operators made by Hansson [37,39], credibility-limited revision operators are placed in the class of *Decision + Revision* operators. This kind of operation encompasses two steps: (1) Decide whether to fully accept, partially accept, or reject the input. (2) Revise when appropriate. Another operator of revision in this category is *screened revision*, proposed by Makinson in [48]. In that paper, a set A of sentences that are immune to revision is introduced. The outcome of revising by sentences that contradict $\mathbf{K} \cap A$ is identical to the original belief set. If the input sentence is compatible with $\mathbf{K} \cap A$, then the belief set is revised essentially in the AGM way. Formally, screened revision for a belief set $\mathbf{K}$ is defined as follows:

$$\mathbf{K}\#_A\alpha = \begin{cases} \mathbf{K} * \alpha & \text{if } \alpha \text{ is consistent with } \mathbf{K} \cap A. \\ \mathbf{K} & \text{otherwise} \end{cases}$$

where $*$ is an AGM revision function with the additional constraint that for all α, $\mathbf{K} \cap A \subseteq \mathbf{K} * \alpha$.

A more general approach, called *generalized screened revision*, was proposed in [37]:

$$\mathbf{K}\#_f\alpha = \begin{cases} \mathbf{K} * \alpha & \text{if } \alpha \text{ is consistent with } \mathbf{K} \cap f(\alpha). \\ \mathbf{K} & \text{otherwise} \end{cases}$$

where f is a function such that for each sentence α, $f(\alpha)$ is a set of sentences. $*$ is a (modified) AGM revision function such that for all α, $\mathbf{K} \cap f(\alpha) \subseteq \mathbf{K} * \alpha$. Different properties can be imposed on f. Makinson [48] proposed, for example, $f(p) = \{q : p < q\}$, where $<$ is a binary relation on the language. Credibility-limited revision operators are considered a generalization of screened revision operators (see [40, Definition 2]). Another non-prioritized revision operator is selective revision, proposed by Fermé and Hansson in [10]. This operator allows the acceptance of only part of the new information and the rejection of the rest of it. An operator of selective revision, $\circledast$, is constructed from a basic AGM revision $*$ and a function f from $\mathcal{L}$ to $\mathcal{L}$ as follows:

$$\mathbf{K} \circledast \alpha = \mathbf{K} \star f(\alpha).$$

Intuitively, f selects the credible part of every sentence. In [19], the adaptation of selective revision operators to the belief base context was studied, and several representation theorems for selective base revision operators were presented.

(c) In addition to shielded contraction, screened revision and credibility-limited revision, there are few works that assign a status to the beliefs regarding their behavior in the revision/contraction process. Among them, Ghose and Goebel ([28]) introduced *explicit disbeliefs* and *explicit beliefs* as allowable epistemic inputs. The explicit disbeliefs are used to record and store sentences that an agent refuses to commit to. Based on this work, Chopra, Ghose and Meyer defined *information states* [7]. An information state I is a finite set of information, *i.e.*, beliefs and disbeliefs. The belief state consisting of all the beliefs in I is denoted by BI. Similarly, the disbelief state consisting of all the disbeliefs in I is denoted by DI. It is important to mention the notion of consistency. An information state is consistent if the beliefs (as a whole) do not imply the negation of any of its disbeliefs. This notion of consistency, which differs from the notion used in credibility-limited revision and shielded contraction, allows an information state to contain both α and $\neg\alpha$ as disbeliefs.

Other works that attribute different statuses to beliefs were presented in [18] and [4]. In these papers, operators similar to credibility-limited revision operators were defined, but different degrees of credibility were considered. The operators designated in [18] by two credibility-limited revisions have the following behavior: a belief at the highest level of credibility is always incorporated when revising by it; if it is considered to be at the second level of credibility, then that sentence is not incorporated in the revision process, but its negation is removed from the original belief set; when revising by a non-credible sentence, the operator leaves the original belief set unchanged. Formally, this operator is defined by means of an AGM revision $*$ and two sets of sentences C_H and C_L as follows:

$$\mathbf{K} \odot \alpha = \begin{cases} \mathbf{K} * \alpha & \text{if } \alpha \in C_H \\ (\mathbf{K} * \alpha) \cap \mathbf{K} & \text{if } \alpha \in C_L \\ \mathbf{K} & \text{if } \alpha \notin (C_L \cup C_H) \end{cases}$$

Note that, according to the Harper identity (and the extensionality of $*$), $\mathbf{K} \div \neg\alpha = (\mathbf{K} * \alpha) \cap \mathbf{K}$, where $\div$ is an AGM contraction operator. The intuition regarding a sentence in C_L, is that it is not credible enough to be incorporated when revising by it but creates in the agent sufficient doubt that forces him or her to remove from the belief set **K** the beliefs that are inconsistent with it.

7. Conclusion

A crucial point in artificial intelligence is the representation of knowledge and its dynamics. Several issues must be overcome when updating the knowledge base with entries that are not consistent with its content. The AGM model provides a conceptual mechanism concerning how to proceed when new information is received. However, this model always gives priority to new information (incorporating it in the case of revision and removing it in the case of contraction). In many cases, this is not a desirable behavior, as the new information may lack credibility, and sometimes there are beliefs that the agent is not willing to remove (regardless of the information received). For this reason, non-prioritized change operators were considered. Two of those are the shielded contractions and the credibility-limited revisions. Furthermore, since belief sets are not suitable for computational implementations, credibility-limited base revision operators and shielded base contractions are relevant in the field of artificial intelligence. The results presented in this paper highlight a duality between these two types of operations in the sense that they allow us to go back and forth between these two classes of operators.

The identity commonly known as the *Levi identity* (*e.g.*, [24,3]) is the result of the formalization by means of an equation of the procedure identified by Isaac Levi, in [45], as the most intuitive one to obtain a (new) belief set that is acceptable as the result of the revision of a (given) belief set by a sentence α. Such identity provides an explicit way of defining a revision function on a belief set from a (given) contraction function on that same belief set. On the other hand, a widely known way of proceeding conversely, *i.e.*, of defining a contraction by means of a revision, is captured by the identity that is commonly known as the *Harper identity* (*e.g.*, [26]) since the idea underlying that equation was first presented by William Harper in [41].[15]

The relevance of the Levi and Harper identities is emphasized by the fact that they lead to the conclusion that there is a one-to-one correspondence between the class of basic AGM contractions and the class of basic AGM revisions. In fact, this interrelation among those classes is an immediate consequence of the following results originally presented by Peter Gärdenfors in [23,25]:

(a) An operator defined from a basic AGM contraction using the Levi identity is a basic AGM revision.
(b) An operator defined from a basic AGM revision using the Harper identity is a basic AGM contraction.
(c) If we regard the Levi and Harper identities as functions (from the class of all basic AGM contractions to the class of all basic AGM revisions and vice versa, respectively), then they are the inverse (functions) of each other.

The present paper shows that the (natural adaptations of) the Levi and Harper identities are also appropriate in the (more general) setting of non-prioritized revisions and contractions on belief bases. In fact, the main results of Subsection 5.2 highlight that the operators obtained from a shielded contraction using the consistency-preserving Levi identity are credibility-limited revisions and, vice versa, the operators obtained from a credibility-limited revision using the Harper identity are shielded contractions. Furthermore, the last two theorems of Subsection 5.2, which can be seen as a generalization of the results from [23,25] stated (informally) in item (c) above, allow us to conclude that there is a one-to-one correspondence between the classes of shielded contractions considered in [21] and the classes of CL revision studied in [22]. More precisely, the results of Section 5.2 assert that for each class $\mathcal{S} \in \{SPMC, SP-SPMC, CC-SPMC, SP+CC-SPMC, P-SPMC, SKC, SP-SKC, CC-SKC, SP+CC-SKC, P-SKC, SSKC, SP-SSKC, CC-SSKC, SP+CC-SSKC, P-SSKC, SbAGMC, SP-SbAGMC, CC-SbAGMC, SP+CC-SbAGMC, P-SbAGMC\}$ there is one and only one class $\mathcal{C} \in \{CLPMR, SI-CLPMR, DD-CLPMR, SI+DD-CLPMR, P-CLPMR, CLKR, SI-CLKR, DD-CLKR, SI+DD-CLKR, P-CLKR, CLSKR, SI-CLSKR, DD-CLSKR, SI+DD-CLSKR, P-CLSKR, CLbAGMR, SI-CLbAGMR, DD-CLbAGMR, SI+DD-CLbAGMR, P-CLbAGMR\}$, such that:

(i) each one of the shielded contractions in $\mathcal{S}$ can be obtained from one and only one of the CL revisions in $\mathcal{C}$ using the Harper identity.
(ii) each one of the CL revisions in $\mathcal{C}$ can be obtained from one and only one of the shielded contractions in $\mathcal{S}$ using the consistency-preserving Levi identity.

[15] We notice, however, that in the literature such equation is also sometimes referred to as the *Gärdenfors identity* (*e.g.*, [3], [46]) since it has also been proposed by Gärdenfors in [24].

In summary, the set of results presented in this paper provides definite evidence of the adequacy of the consistency-preserving Levi identity and Harper identity as procedures for obtaining credibility-limited base revision operators from shielded base contraction operators and vice versa, respectively. On the other hand, these results also highlight the very strong interconnection among credibility-limited base revisions and shielded base contractions in the sense that these two kinds of non-prioritized belief change operators are interdefinable by means of the consistency-preserving Levi identity and the Harper identity.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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Appendix A. Proofs

Lemma 1. *Let $A = \{p, p \vee q, p \to q\}$, Cn be purely truth-functional and $R = \mathcal{L} \setminus Cn(p \vee q)$. It holds that:*

1. *R satisfies* conjunctive completeness, non-retractability propagation *and* uniform retractability.
2. *If $\div$ is a kernel contraction based on an incision function σ such that $\sigma(\alpha) \subseteq R$, for all sentences $\alpha \in R$, then condition (**R - $\div$**) holds.*

Proof. 1. **Conjunctive completeness:** Let $\alpha \wedge \beta \in R$. Hence $\{p \vee q\} \nvdash \alpha \wedge \beta$. Therefore $\{p \vee q\} \nvdash \alpha$ or $\{p \vee q\} \nvdash \beta$, from which it follows that $\alpha \in R$ or $\beta \in R$.
Non-retractability propagation: Let $\alpha \notin R$. Then $\{p \vee q\} \vdash \alpha$. Let $\beta \in Cn(\alpha)$. Hence $\{p \vee q\} \vdash \beta$, from which it follows that $\beta \notin R$.
Uniform retractability: Assume that it holds for all subsets A' of A that $\alpha \in Cn(A')$ if and only if $\beta \in Cn(A')$. Let $\alpha \notin R$. Hence $\{p \vee q\} \vdash \alpha$, from which it follows that $\{p \vee q\} \vdash \beta$. Thus $\beta \notin R$. It also holds, by symmetry of the case, that if $\beta \notin R$, then $\alpha \notin R$. Hence $\alpha \in R$ if and only if $\beta \in R$.
2. Let $\alpha \notin R$ and $\beta \in R$. Hence $\alpha \in Cn(p \vee q)$ and $\beta \notin Cn(p \vee q)$. It holds that $\sigma(\beta) \cap Cn(p \vee q) = \emptyset$. Therefore $A \div \beta \vdash \alpha$. ■

Lemma 2. *Let $A = \{p, q\}$, Cn be purely truth-functional and C be the set defined by the following condition:*

$\alpha \in C$ *if and only if* $\neg\alpha \notin Cn(p) \cup Cn(q)$

Then C satisfies element consistency, expansive credibility, uniform credibility *and* single sentence closure.

Proof. Element consistency: Let $\alpha \in C$. Hence $\neg\alpha \notin Cn(p) \cup Cn(q)$. Thus $\neg\alpha \notin Cn(\emptyset)$ (since $Cn(\emptyset) \subseteq Cn(p) \cup Cn(q)$). Therefore $\alpha \nvdash \perp$.
Expansive credibility: Assume that $\neg\alpha \notin C$. Hence $\alpha \in Cn(p) \cup Cn(q)$. Therefore $A \vdash \alpha$.
Uniform credibility: Assume that it holds, for all subsets $A' \subseteq A$, that $A' \cup \{\alpha\} \vdash \perp$ if and only if $A' \cup \{\beta\} \vdash \perp$. Let $\alpha \in C$. Hence $\neg\alpha \notin Cn(p) \cup Cn(q)$. Thus $\neg\alpha \notin Cn(p)$ and $\neg\alpha \notin Cn(q)$. Therefore $\{p, \alpha\} \nvdash \perp$ and $\{q, \alpha\} \nvdash \perp$. Thus, by hypothesis, $\{p, \beta\} \nvdash \perp$ and $\{q, \beta\} \nvdash \perp$. Hence $\neg\beta \notin Cn(p)$ and $\neg\beta \notin Cn(q)$, from which it follows that $\neg\beta \notin Cn(p) \cup Cn(q)$. Thus $\beta \in C$. By symmetry of the case it follows that if $\beta \in C$, then $\alpha \in C$. Therefore it holds that $\alpha \in C$ if and only if $\beta \in C$.
Single sentence closure: Let $\alpha \in C$ and $\beta \in Cn(\alpha)$. Thus $\neg\alpha \notin Cn(p) \cup Cn(q)$ and $\{\neg\beta\} \vdash \neg\alpha$. Therefore $\neg\beta \notin Cn(p) \cup Cn(q)$. Hence $\beta \in C$. ■

Lemma 3. *Let R and C be subsets of $\mathcal{L}$. If R and C are closed under double negation, then condition (**C-R**) holds if and only if condition (**R-C**) also holds.*

Proof of Lemma 3. Let R and C be subsets of $\mathcal{L}$ that are closed under double negation. We intend to prove that condition (**C-R**) holds if and only if condition (**R-C**) also holds.
(From left to right) $\alpha \in R$ iff $\neg\neg\alpha \in R$ iff $\neg\alpha \in C$.
(From right to left) $\alpha \in C$ iff $\neg\neg\alpha \in C$ iff $\neg\alpha \in R$. ■

Proof of Observation 5.1. Let R and C be subsets of $\mathcal{L}$. We intend to prove that R is closed under double negation and condition (**C-R**) holds if and only if C is closed under double negation and condition (**R-C**) holds.
(From left to right) $\alpha \in C$ iff $\neg\alpha \in R$ iff $\neg\neg\neg\alpha \in R$ iff $\neg\neg\alpha \in C$. Therefore C is closed under double negation. Thus, according to Lemma 3, condition (**R-C**) holds.
(From right to left) $\alpha \in R$ iff $\neg\alpha \in C$ iff $\neg\neg\neg\alpha \in C$ iff $\neg\neg\alpha \in R$. Therefore R is closed under double negation. Thus, according to Lemma 3, condition (**C-R**) holds. ■

Proof of Observation 5.2. Let A be a belief base, R and C be sets of sentences that are closed under double negation and satisfy conditions (**C-R**) and (**R-C**).

(a) Let R be a set that satisfies *non-retractability of logical equivalents* we intend to prove that C satisfies *credibility of logical equivalents*.
Let $\vdash \alpha \leftrightarrow \beta$ and assume without loss of generality that $\alpha \in C$. Hence $\neg\alpha \in R$. Therefore $\neg\beta \in R$, since R satisfies *non-retractability of logical equivalents*. Thus $\beta \in C$. By symmetry of the case it follows that if $\beta \in C$, then $\alpha \in C$. Thus $\alpha \in C$ if and only if $\beta \in C$.
Let C be a set that satisfies *credibility of logical equivalents* we intend to prove that R satisfies *non-retractability of logical equivalents*.
Let $\vdash \alpha \leftrightarrow \beta$ and assume without loss of generality that $\alpha \in R$. Hence $\neg\neg\alpha \in R$, from which it follows that $\neg\alpha \in C$. Therefore $\neg\beta \in C$, since C satisfies *credibility of logical equivalents* (and $\vdash \neg\alpha \leftrightarrow \neg\beta$). Thus $\neg\neg\beta \in R$, from which it follows that $\beta \in R$. By symmetry of the case it follows that if $\beta \in R$, then $\alpha \in R$. Thus $\alpha \in R$ if and only if $\beta \in R$.
Let R be a set that satisfies *non-retractability of tautology* we intend to prove that C satisfies *element consistency*.
Let $\alpha \vdash \bot$. Hence $\vdash \neg\alpha$. Therefore $\neg\alpha \notin R$, since R satisfies *non-retractability of tautology*. Thus $\alpha \notin C$.
Let C be a set that satisfies *element consistency* we intend to prove that R satisfies *non-retractability of tautology*.
Let $\vdash \alpha$. Hence $\{\neg\alpha\} \vdash \bot$. Thus, since C satisfies *element consistency*, $\neg\alpha \notin C$. From which it follows that $\neg\neg\alpha \notin R$. Therefore $\alpha \notin R$.
Let R be a set that satisfies *non-retractability propagation* we intend to prove that C satisfies *single sentence closure*.
Let $\alpha \in C$ and $\beta \in Cn(\alpha)$. Hence $\neg\alpha \in R$ and, by deduction, $\vdash \alpha \to \beta$. Thus $\vdash \neg\beta \to \neg\alpha$. Therefore $\neg\alpha \in Cn(\neg\beta)$. Assume by *reductio ad absurdum* that $\beta \notin C$. Hence $\neg\beta \notin R$. From which it follows that $\neg\alpha \notin R$, since R satisfies *non-retractability propagation*. Contradiction. Thus $\beta \in C$.
Let C be a set of sentences that satisfies *single sentence closure* we intend to prove that R satisfies *non-retractability propagation*.
Let $\alpha \notin R$ and suppose that $\beta \in Cn(\alpha)$. From $\alpha \notin R$ it follows that $\neg\neg\alpha \notin R$, from which it follows that $\neg\alpha \notin C$. From $\beta \in Cn(\alpha)$ it follows that $\vdash \neg\beta \to \neg\alpha$. Thus by *single sentence closure*, it follows that $\neg\beta \notin C$. Therefore $\neg\neg\beta \notin R$, from which it follows, that $\beta \notin R$.
Let R be a set that satisfies *uniform retractability with respect to* A. We intend to prove that C satisfies *uniform credibility with respect to* A.
Assume that it holds for all subsets A' of A that $A' \vdash \neg\alpha$ if and only if $A' \vdash \neg\beta$. Then, $\neg\alpha \in R$ if and only if $\neg\beta \in R$, since R satisfies *uniform retractability* with respect to A. Hence $\alpha \in C$ if and only if $\beta \in C$.
Let C be a set that satisfies *uniform credibility with respect to* A. We intend to prove that R satisfies *uniform retractability with respect to* A.
Assume that it holds for all subsets A' of A that $A' \vdash \alpha$ if and only if $A' \vdash \beta$. Hence for all subsets A' of A it holds that $A' \cup \{\neg\alpha\} \vdash \bot$ if and only if $A' \cup \{\neg\beta\} \vdash \bot$. Therefore, by *uniform credibility*, it follows that $\neg\alpha \in C$ if and only if $\neg\beta \in C$. Thus $\neg\neg\alpha \in R$ if and only if $\neg\neg\beta \in R$. Hence $\alpha \in R$ if and only if $\beta \in R$.
Let R be a set that satisfies *non-retractability upper bounding with respect to* A. We intend to prove that C satisfies *expansive credibility with respect to* A. Assume that $\neg\alpha \notin C$. Hence $\neg\neg\alpha \notin R$, from which it follows that $\alpha \notin R$. Thus $A \vdash \alpha$, since by *non-retractability upper bounding*, $\mathcal{L} \setminus R \subseteq Cn(A)$.
Let C be a set that satisfies *expansive credibility with respect to* A. We intend to prove that R satisfies *non-retractability upper bounding with respect to* A.
Let $\alpha \in \mathcal{L} \setminus R$. Then $\neg\neg\alpha \notin R$, from which it follows that $\neg\alpha \notin C$. Hence, by *expansive credibility with respect to* A, it follows that $A \vdash \alpha$.

b) Assume that R satisfies *retractability of logical equivalents* and C satisfies *credibility of logical equivalents*.
Let R be a set that satisfies *conjunctive completeness* we intend to prove that C satisfies *disjunctive completeness*.
Let $\alpha \notin C$ and $\beta \notin C$. Therefore $\neg\alpha \notin R$ and $\neg\beta \notin R$. By *conjunctive completeness* it follows that $\neg\alpha \wedge \neg\beta \notin R$. Thus, by *retractability of logical equivalents*, it follows that $\neg(\alpha \vee \beta) \notin R$. Therefore $\alpha \vee \beta \notin C$.
Let C be a set that satisfies *disjunctive completeness* we intend to prove that R satisfies *conjunctive completeness*.
Let $\alpha \notin R$ and $\beta \notin R$. Thus $\neg\neg\alpha \notin R$ and $\neg\neg\beta \notin R$, from which it follows that $\neg\alpha \notin C$ and $\neg\beta \notin C$. From which it

follows, by *disjunctive completeness* that $\neg\alpha \vee \neg\beta \notin C$. Hence $\neg(\neg\alpha \vee \neg\beta) \notin R$. Thus by *retractability of logical equivalents* it follows that $\alpha \wedge \beta \notin R$. ■

Proof of Observation 5.3. Let $\alpha \notin C$ and $\beta \in C$. We intent to prove that $A \cap A * \beta \vdash \neg\alpha$.
From $\alpha \notin C$ and $\beta \in C$ it follows by condition (**C-R**) that $\neg\alpha \notin R$ and $\neg\beta \in R$. Therefore, according to condition (**R - $\div$**), $A \div \neg\beta \vdash \neg\alpha$. On the other hand, by the Levi identity, $A * \beta = (A \div \neg\beta) \cup \{\beta\}$. Therefore, $A \cap A * \beta = A \cap ((A \div \neg\beta) \cup \{\beta\}) = (A \cap (A \div \neg\beta)) \cup (A \cap \{\beta\}) = (A \div \neg\beta) \cup (A \cap \{\beta\})$. Thus $A \div \neg\beta \subseteq A \cap A * \beta$, from which it follows that $A \cap A * \beta \vdash \neg\alpha$. ■

Proof of Observation 5.4. Let $\alpha \notin R$ and $\beta \in R$. We intent to prove that $A \div \beta \vdash \alpha$.
By condition (**R-C**), it follows that $\neg\alpha \notin C$ and $\neg\beta \in C$. Therefore, according to condition (**C - $*$**), $A \cap A * \neg\beta \vdash \neg\neg\alpha$. Thus, it follows from the Harper identity that $A \div \beta \vdash \alpha$. ■

Proof of Theorem 5.5. Assume first that $\alpha \in R$. It follows that $\neg\alpha \in C$. Hence $A \sim \alpha = A \div \alpha = (A * \neg\alpha) \cap A = (A \circledast \neg\alpha) \cap A$. If $\alpha \notin R$, then $\neg\alpha \notin C$. Hence $A \circledast \neg\alpha = A$ and $A \sim \alpha = A$. Thus $A \sim \alpha = A = (A \circledast \neg\alpha) \cap A$. ■

Proof of Theorem 5.6. (a) Assume first that $\alpha \notin C$. Hence $A \circledast \alpha = A$. If $\alpha \in C$, then $\neg\alpha \in R$. Thus $A \circledast \alpha = A * \alpha = (A \div \neg\alpha) \cup \{\alpha\} = (A \sim \neg\alpha) \cup \{\alpha\}$.

(b) It remains to prove that: $\alpha \in C$ if and only if $A \sim \neg\alpha \nvdash \neg\alpha$.
Let $\alpha \notin C$. Then $\neg\alpha \notin R$. Hence, by $\sim$ definition, $A \sim \neg\alpha = A$. Thus, by R *non-retractability upper bounding*, $A \sim \neg\alpha \vdash \neg\alpha$.
Let $\alpha \in C$. Then $\neg\alpha \in R$. Therefore, by $\sim$ definition, $A \sim \neg\alpha = A \div \neg\alpha$. On the other hand, by R *non-retractability of tautology*, $\nvdash \neg\alpha$. Thus, by $\div$ *success*, $A \sim \neg\alpha \nvdash \neg\alpha$. ■

Proof of Theorem 5.7. Let A be a consistent belief base and (for all $\alpha \in \mathcal{L}$)

$$A \circledast \alpha = \begin{cases} (A \sim \neg\alpha) \cup \{\alpha\} & \text{if } A \sim \neg\alpha \nvdash \neg\alpha \\ A & \text{otherwise} \end{cases}$$

Relative success and *consistency preservation* follow directly from the definition of $\circledast$.

Assume that $\sim$ satisfies *inclusion*. We intend to prove that $\circledast$ satisfies *inclusion*. It follows directly from the definition of $\circledast$ and $\sim$ *inclusion* that $A \circledast \alpha \subseteq A \cup \{\alpha\}$.

Assume that $\sim$ satisfies *inclusion* and *persistence*. We intend to prove that $\circledast$ satisfies *disjunctive distribution* and *persistence*.

Disjunctive distribution: Let $\alpha \notin A \circledast \alpha$ and $\beta \notin A \circledast \beta$. Hence, by definition of $\circledast$, $A \circledast \alpha = A \circledast \beta = A$. Furthermore, $A \sim \neg\alpha \vdash \neg\alpha$ and $A \sim \neg\beta \vdash \neg\beta$. By $\sim$ *persistence* $A \sim \neg(\alpha \vee \beta) \vdash \neg\alpha$ and $A \sim \neg(\alpha \vee \beta) \vdash \neg\beta$. Hence $A \sim \neg(\alpha \vee \beta) \vdash \neg(\alpha \vee \beta)$. Hence $A \circledast (\alpha \vee \beta) = A$. Thus $\alpha \vee \beta \notin A \circledast (\alpha \vee \beta)$, since $A \nvdash \bot$ and by $\sim$ *inclusion* $A \vdash \neg(\alpha \vee \beta)$.

Persistence: Let $A \cap A \circledast \beta \vdash \neg\beta$. Hence $A \vdash \neg\beta$ and $A \circledast \beta \vdash \neg\beta$. If $A \sim \neg\beta \nvdash \neg\beta$, then $\beta \in A \circledast \beta$. Thus $A \circledast \beta \vdash \bot$, from which it follows, by $\circledast$ definition and deduction, that $A \sim \neg\beta \vdash \neg\beta$. Contradiction. Hence $A \sim \neg\beta \vdash \neg\beta$. From which it follows, by $\sim$ *persistence*, that $A \sim \neg\alpha \vdash \neg\beta$. By $\sim$ *inclusion* it follows that $A \sim \neg\alpha \subseteq A$. We will consider two cases:
Case 1) $A \sim \neg\alpha \vdash \neg\alpha$. Hence $A \circledast \alpha = A$. Then $A \cap A \circledast \alpha = A \vdash \neg\beta$.
Case 2) $A \sim \neg\alpha \nvdash \neg\alpha$. Hence $A \circledast \alpha = (A \sim \neg\alpha) \cup \{\alpha\}$ from which it follows that $A \cap A \circledast \alpha = A \cap ((A \sim \neg\alpha) \cup \{\alpha\}) = (A \cap (A \sim \neg\alpha)) \cup (A \cap \{\alpha\}) = (A \sim \neg\alpha) \cup (A \cap \{\alpha\})$. Thus $A \sim \neg\alpha \subseteq A \cap A \circledast \alpha$. Therefore $A \cap A \circledast \alpha \vdash \neg\beta$.

Assume that $\sim$ satisfies *inclusion*, *vacuity* and *uniformity*. We intend to prove that $\circledast$ satisfies *uniformity*.
Let it be the case that for all subsets A' of A, $A' \cup \{\alpha\} \vdash \bot$ if and only if $A' \cup \{\beta\} \vdash \bot$. Hence, for all $A' \subseteq A$, $A' \vdash \neg\alpha$ if and only if $A' \vdash \neg\beta$. By $\sim$ *uniformity* $A \sim \neg\alpha = A \sim \neg\beta$. By $\sim$ *inclusion* $A \sim \neg\beta \subseteq A$.
If $A \sim \neg\alpha \vdash \neg\alpha$. Then $A \sim \neg\beta \vdash \neg\alpha$. Thus, by hypothesis, $A \sim \neg\beta \vdash \neg\beta$. Hence, by definition of $\circledast$, $A \circledast \alpha = A \circledast \beta = A$. Therefore $A \cap A \circledast \alpha = A \cap A \circledast \beta$.
If $A \sim \neg\alpha \nvdash \neg\alpha$. Then $A \sim \neg\beta \nvdash \neg\alpha$. From which it follows, by hypothesis, that $A \sim \neg\beta \nvdash \neg\beta$. Therefore, by definition of $\circledast$, $A \circledast \alpha = (A \sim \neg\alpha) \cup \{\alpha\}$ and $A \circledast \beta = (A \sim \neg\beta) \cup \{\beta\}$.
There are three cases to consider:
Case 1) $\alpha \in A$. Then $A \nvdash \neg\alpha$ (since $A \nvdash \bot$). Hence, by $\sim$ *vacuity* and *inclusion* $A \sim \neg\alpha = A$. Thus $A \sim \neg\beta = A$. Therefore $A \cap A \circledast \alpha = A \cap ((A \sim \neg\alpha) \cup \{\alpha\}) = A \cap (A \cup \{\alpha\}) = A$. Using a similar reasoning, it follows that $A \cap A \circledast \beta = A$. Hence $A \cap A \circledast \alpha = A \cap A \circledast \beta$.
Case 2) $\beta \in A$. This case is similar to the previous one.
Case 3) $\alpha \notin A$ and $\beta \notin A$. Then $A \cap A \circledast \alpha = A \cap ((A \sim \neg\alpha) \cup \{\alpha\}) = (A \cap A \sim \neg\alpha) \cup (A \cap \{\alpha\}) = A \sim \neg\alpha = A \sim \neg\beta = (A \cap A \sim \neg\beta) \cup (A \cap \{\beta\}) = A \cap ((A \sim \neg\beta) \cup \{\beta\}) = A \cap A \circledast \beta$.

Assume that $\sim$ satisfies *relevance*. We intend to prove that $\circledast$ satisfies *relevance*.
Let $\beta \in A$ and $\beta \notin A \circledast \alpha$. Hence $A \neq A \circledast \alpha$. Thus $A \circledast \alpha = (A \sim \neg\alpha) \cup \{\alpha\}$ and $A \sim \neg\alpha \nvdash \neg\alpha$. Hence $\beta \notin A \sim \neg\alpha$. By $\sim$ *relevance* there is some A' such that $A \sim \neg\alpha \subseteq A' \subseteq A$, $A' \nvdash \neg\alpha$ but $A' \cup \{\beta\} \vdash \neg\alpha$. Let $X = A' \cup \{\alpha\}$. Thus $A \circledast \alpha \subseteq X \subseteq A \cup \{\alpha\}$.
It remains to prove that:
1) $X \nvdash \bot$.
2) $X \cup \{\beta\} \vdash \bot$.

1) Assume by *reductio ad absurdum* that $X \vdash \perp$. Hence $A' \cup \{\alpha\} \vdash \perp$. Thus, by deduction, $A' \vdash \alpha \rightarrow \perp$. Hence $A' \vdash \neg\alpha$. Contradiction.
2) $X \cup \{\beta\} = A' \cup \{\alpha, \beta\}$. Since $A' \cup \{\beta\} \vdash \neg\alpha$ it follows that $A' \cup \{\alpha, \beta\} \vdash \perp$. Hence $X \cup \{\beta\} \vdash \perp$.

Assume that $\sim$ satisfies *core-retainment*. We intend to prove that $\circledast$ satisfies *core-retainment*.
Let $\beta \in A$ and $\beta \notin A \circledast \alpha$. Hence $A \neq A \circledast \alpha$. Thus $A \circledast \alpha = (A \sim \neg\alpha) \cup \{\alpha\}$ and $A \sim \neg\alpha \nvdash \neg\alpha$. Hence $\beta \notin A \sim \neg\alpha$. By $\sim$ *core-retainment* there is some A' such that $A' \subseteq A$, $A' \nvdash \neg\alpha$ but $A' \cup \{\beta\} \vdash \neg\alpha$.

Assume that $\sim$ satisfies *conjunctive constancy, relative success* and *extensionality*. We intend to prove that $\circledast$ satisfies *disjunctive distribution*.
Let $\alpha \notin A \circledast \alpha$ and $\beta \notin A \circledast \beta$. By definition of $\circledast$ it follows that $A \sim \neg\alpha \vdash \neg\alpha$ and $A \sim \neg\beta \vdash \neg\beta$. Hence by $\sim$ *relative success*, $A \sim \neg\alpha = A \sim \neg\beta = A$. Thus, by $\sim$ *conjunctive constancy*, $A \sim (\neg\alpha \wedge \neg\beta) = A$. From which it follows, by $\sim$ *extensionality*, that $A \sim \neg(\alpha \vee \beta) = A$. On the other hand $A \vdash \neg\alpha \wedge \neg\beta$. Hence $A \sim \neg(\alpha \vee \beta) \vdash \neg(\alpha \vee \beta)$. Therefore, by definition of $\circledast$, $A \circledast (\alpha \vee \beta) = A$. From $A \nvdash \perp$ it follows that $A \circledast (\alpha \vee \beta) \nvdash \alpha \vee \beta$. Hence $\alpha \vee \beta \notin A \circledast (\alpha \vee \beta)$.

Assume that $\sim$ satisfies *inclusion* and *success propagation*. We intend to prove that $\circledast$ satisfies *strict improvement*.
Let $\alpha \in A \circledast \alpha$ and $\vdash \alpha \rightarrow \beta$. Hence $\vdash \neg\beta \rightarrow \neg\alpha$. Assume by *reductio ad absurdum* that $A \sim \neg\alpha \vdash \neg\alpha$. Then, by $\sim$ *inclusion*, $A \vdash \neg\alpha$. On the other hand, it follows from $\circledast$ definition that $A \circledast \alpha = A$. Hence $\alpha \in A$. Therefore $A \vdash \perp$. Contradiction. Hence $A \sim \neg\alpha \nvdash \neg\alpha$. Thus, by $\sim$ *success propagation*, $A \sim \neg\beta \nvdash \neg\beta$. From which it follows, by definition of $\circledast$, that $\beta \in A \circledast \beta$.

Assume that $\sim$ satisfies *inclusion* and *vacuity*. We intend to prove that $\circledast$ satisfies *vacuity*.
Assume that $A \nvdash \neg\alpha$. By $\sim$ *inclusion* and *vacuity* it follows that $A \sim \neg\alpha = A \nvdash \neg\alpha$. Hence, by definition of $\circledast$, it follows that $A \circledast \alpha = (A \sim \neg\alpha) \cup \{\alpha\} = A \cup \{\alpha\}$.

Assume that $\sim$ satisfies *disjunctive elimination*. We intend to prove that $\circledast$ satisfies *disjunctive elimination*.
Let $\beta \in A$ and $\beta \notin A \circledast \alpha$. Then $A \circledast \alpha \neq A$. Thus $A \sim \neg\alpha \nvdash \neg\alpha$ and $A \circledast \alpha = (A \sim \neg\alpha) \cup \{\alpha\}$. Thus $\beta \notin A \sim \neg\alpha$, from which it follows by *disjunctive elimination* that $A \sim \neg\alpha \nvdash \neg\alpha \vee \beta$. Hence $(A \sim \neg\alpha) \cup \{\alpha\} \nvdash \neg\alpha \vee \beta$, otherwise it would follow by deduction that $A \sim \neg\alpha \vdash \alpha \rightarrow (\neg\alpha \vee \beta)$ and consequently that $A \sim \neg\alpha \vdash \neg\alpha \vee \beta$, since $\vdash (\neg\alpha \vee \beta) \leftrightarrow (\alpha \rightarrow (\neg\alpha \vee \beta))$. Therefore $A \circledast \alpha \nvdash \neg\alpha \vee \beta$.

Assume that $\sim$ satisfies *extensionality, inclusion* and *vacuity*. We intend to prove that $\circledast$ satisfies *weak extensionality*.
Let $\vdash \alpha \leftrightarrow \beta$. Then $\vdash \neg\alpha \leftrightarrow \neg\beta$. We will prove by cases:
Case 1) $A \sim \neg\alpha \vdash \neg\alpha$. Then $A \sim \neg\alpha \vdash \neg\beta$. Thus, by $\sim$ *extensionality*, $A \sim \neg\beta \vdash \neg\beta$. Therefore, by definition of $\circledast$, $A \circledast \alpha = A \circledast \beta = A$. Hence $A \cap A \circledast \alpha = A \cap A \circledast \beta$.
Case 2) $A \sim \neg\alpha \nvdash \neg\alpha$. Then $A \sim \neg\alpha \nvdash \neg\beta$. Therefore, by $\sim$ *extensionality*, $A \sim \neg\beta \nvdash \neg\beta$. Therefore, by definition of $\circledast$, $A \circledast \alpha = (A \sim \neg\alpha) \cup \{\alpha\}$ and $A \circledast \beta = (A \sim \neg\beta) \cup \{\beta\}$.
Case 2.1) $\alpha \in A$. Therefore $A \nvdash \neg\alpha$ (since $A \nvdash \perp$) and $A \nvdash \neg\beta$. By $\sim$ *vacuity* and *inclusion* it follows that $A \sim \neg\alpha = A \sim \neg\beta = A$. Hence $A \cap A \circledast \alpha = A \cap ((A \sim \neg\alpha) \cup \{\alpha\}) = A$. By symmetry of the case, it holds that $A \cap A \circledast \beta = A$. Therefore $A \cap A \circledast \alpha = A \cap A \circledast \beta$.
Case 2.2) $\beta \in A$. Follows as in the previous case.
Case 2.3) $\alpha \notin A$ and $\beta \notin A$. By $\sim$ *inclusion* it follows that $A \sim \neg\alpha \subseteq A$ and $A \sim \neg\beta \subseteq A$. Hence $A \cap A \circledast \alpha = A \cap ((A \sim \neg\alpha) \cup \{\alpha\}) = (A \cap A \sim \neg\alpha) \cup (A \cap \{\alpha\}) = A \sim \neg\alpha$. By symmetry of the case, it follows that $A \cap A \circledast \beta = A \sim \neg\beta$. Hence by $\sim$ *extensionality* it follows that $A \cap A \circledast \alpha = A \cap A \circledast \beta$.

Assume that $\sim$ satisfies *inclusion, vacuity* and *relative closure*. We intend to prove that $\circledast$ satisfies *weak relative closure*.
Let $\beta \in A \cap Cn(A \cap A \circledast \alpha)$. It follows trivially if $A \sim \neg\alpha \vdash \neg\alpha$. Assume now that $A \sim \neg\alpha \nvdash \neg\alpha$. Hence $A \circledast \alpha = (A \sim \neg\alpha) \cup \{\alpha\}$. Hence $\beta \in A \cap Cn(A \cap ((A \sim \neg\alpha) \cup \{\alpha\}))$. Hence $\beta \in A$ and $\beta \in Cn((A \cap (A \sim \neg\alpha)) \cup (A \cap \{\alpha\}))$. We will prove by cases:
Case 1) $\alpha \in A$. Then $A \nvdash \neg\alpha$ (since $A \nvdash \perp$). Therefore, by $\sim$ *inclusion* and *vacuity*, it follows that $A \sim \neg\alpha = A$. Thus $A \circledast \alpha = A \cup \{\alpha\}$. Hence $\beta \in A \circledast \alpha$.
Case 2) $\alpha \notin A$. Thus $\beta \in A$ and $\beta \in Cn((A \sim \neg\alpha) \cap A)$. By $\sim$ *inclusion* it follows that $\beta \in A$ and $\beta \in Cn(A \sim \neg\alpha)$. Therefore, by $\sim$ *relative closure* it follows that $\beta \in A \sim \neg\alpha$. Thus $\beta \in A \circledast \alpha$. ■

Proof of Theorem 5.8. Let A be a consistent belief base and $A \sim \alpha = A \cap A \circledast \neg\alpha$ (for all $\alpha \in \mathcal{L}$).
That $\sim$ satisfies *inclusion* follows directly from the definition of $\sim$.

Assume that $\circledast$ satisfies *relative success* and *consistency preservation*. We intend to prove that $\sim$ satisfies *relative success*.
If $A \sim \alpha \vdash \alpha$, then by definition of $\sim$, $A \circledast \neg\alpha \vdash \alpha$. Hence, by $\circledast$ *consistency preservation*, $\neg\alpha \notin A \circledast \neg\alpha$. Thus, by $\circledast$ *relative success*, $A \circledast \neg\alpha = A$. Therefore $A \sim \alpha = A$.

Assume that $\circledast$ satisfies *persistence*. We intend to prove that $\sim$ satisfies *persistence*.
Let $A \sim \alpha \nvdash \beta$. Then $A \cap A \circledast \neg\alpha \nvdash \beta$. Thus, by $\circledast$ *persistence*, $A \cap A \circledast \neg\beta \nvdash \beta$. Therefore, $A \sim \beta \nvdash \beta$.

Assume that $\circledast$ satisfies *relative success* and *relevance*. We intend to prove that $\sim$ satisfies *relevance*.
Let $\beta \in A$ and $\beta \notin A \sim \alpha$. Then, by definition of $\sim$, $\beta \notin A \circledast \neg\alpha$. Thus $A \circledast \neg\alpha \neq A$. By $\circledast$ *relative success* it follows that $\neg\alpha \in A \circledast \neg\alpha$. By $\circledast$ *relevance*, there is some A' such that $A \circledast \neg\alpha \subseteq A' \subseteq A \cup \{\neg\alpha\}$, $A' \nvdash \perp$ but $A' \cup \{\beta\} \vdash \perp$. Let $X = A' \setminus \{\neg\alpha\}$. Hence $X \subseteq A$ and, since $\neg\alpha \in A \circledast \neg\alpha \subseteq A'$, it follows that $X \cup \{\neg\alpha\} = A'$. Therefore $A \circledast \neg\alpha \subseteq X \cup \{\neg\alpha\}$. Thus $A \sim \alpha \subseteq X \cup \{\neg\alpha\}$. To prove that $A \sim \alpha \subseteq X$ it is enough to show that $A \sim \alpha \nvdash \neg\alpha$.
Assume by *reductio ad absurdum* that $A \sim \alpha \vdash \neg\alpha$. Hence, by definition of $\sim$, $A \vdash \neg\alpha$. From $A' \subseteq A \cup \{\neg\alpha\}$ and $A' \cup \{\beta\} \vdash \perp$ it follows, by monotony, that $A \cup \{\neg\alpha, \beta\} \vdash \perp$. From $\beta \in A$ if follows that $A \cup \{\neg\alpha\} \vdash \perp$. Thus from $A \vdash \neg\alpha$ it follows that $A \vdash \perp$. Contradiction. Therefore $A \sim \alpha \nvdash \neg\alpha$. Hence $A \sim \alpha \subseteq X$. On the other hand, from $A' \nvdash \perp$, it follows that $X \cup \{\neg\alpha\} \nvdash \perp$. Thus $X \nvdash \alpha$. From $A' \cup \{\beta\} \vdash \perp$ it follows that $X \cup \{\neg\alpha, \beta\} \vdash \perp$. From which it follows, using deduction, that $X \cup \{\beta\} \vdash \alpha$.

Assume that $\circledast$ satisfies *core-retainment*. We intend to prove that $\sim$ satisfies *core-retainment*.
Let $\beta \in A$ and $\beta \notin A \sim \alpha$. Then, by definition of $\sim$, $\beta \notin A \circledast \neg\alpha$. Hence, by $\circledast$ *core-retainment*, there is some A' such that $A' \subseteq A$, $A' \nvdash \alpha$ but $A' \cup \{\beta\} \vdash \alpha$. Hence $\sim$ satisfies *core-retainment*.

Assume that $\circledast$ satisfies *uniformity*. We intend to prove that $\sim$ satisfies *uniformity*.
Let it be the case that for all subsets A' of A, $A' \vdash \alpha$ if and only if $A' \vdash \beta$. Hence, for all subsets A' of A it holds that $A' \cup \{\neg\alpha\} \vdash \bot$ if and only if $A' \cup \{\neg\beta\} \vdash \bot$. Therefore, by $\circledast$ *uniformity*, $A \cap A \circledast \neg\alpha = A \cap A \circledast \neg\beta$. Thus, by definition of $\sim$, $A \sim \alpha = A \sim \beta$.

Assume that $\circledast$ satisfies *vacuity*. We intend to prove that $\sim$ satisfies *vacuity*.
Assume that $A \nvdash \alpha$. Hence $A \nvdash \neg\neg\alpha$. Thus, by $\circledast$ *vacuity* it follows that $A \cup \{\neg\alpha\} \subseteq A \circledast \neg\alpha$. Thus $A \subseteq A \circledast \neg\alpha$. Therefore $A \sim \alpha = (A \circledast \neg\alpha) \cap A = A$.

Assume that $\circledast$ satisfies *vacuity, relative success, consistency preservation, disjunctive distribution* and *weak extensionality*. We intend to prove that $\sim$ satisfies *conjunctive constancy*. As shown above $\sim$ satisfies *inclusion* and *vacuity*.
Let $A \sim \alpha = A \sim \beta = A$. We will consider three cases:
Case 1) $A \circledast \neg\alpha \nvdash \alpha$. Then $A \sim \alpha \nvdash \alpha$. Thus $A \nvdash \alpha$. From which it follows that $A \nvdash \alpha \wedge \beta$. Hence, by $\sim$ *inclusion* and *vacuity*, $A \sim (\alpha \wedge \beta) = A$.
Case 2) $A \circledast \neg\beta \nvdash \beta$. This case is symmetrical with the first case.
Case 3) $A \circledast \neg\alpha \vdash \alpha$ and $A \circledast \neg\beta \vdash \beta$. By $\circledast$ *consistency preservation* it follows that $\neg\alpha \notin A \circledast \neg\alpha$ and $\neg\beta \notin A \circledast \neg\beta$. Hence, by *disjunctive distribution*, it follows that $\neg\alpha \vee \neg\beta \notin A \circledast (\neg\alpha \vee \neg\beta)$. Hence, by $\circledast$ *relative success*, $A \circledast (\neg\alpha \vee \neg\beta) = A$. By definition of $\sim$, $A \sim (\alpha \wedge \beta) = A \cap A \circledast \neg(\alpha \wedge \beta)$. Thus, by $\circledast$ *weak extensionality*, it follows that $A \sim (\alpha \wedge \beta) = A \cap A \circledast (\neg\alpha \vee \neg\beta) = A$.

Assume that $\circledast$ satisfies *disjunctive elimination*. We intend to prove that $\sim$ satisfies *disjunctive elimination*.
Let $\beta \in A$ and $\beta \notin A \sim \alpha$. By definition of $\sim$ it follows that $\beta \notin A \circledast \neg\alpha$. Therefore, by $\circledast$ *disjunctive elimination*, $A \circledast \neg\alpha \nvdash (\neg\neg\alpha) \vee \beta$. Thus $A \circledast \neg\alpha \nvdash \alpha \vee \beta$. Therefore $A \sim \alpha \nvdash \alpha \vee \beta$.

Assume that $\circledast$ satisfies *weak extensionality*. We intend to prove that $\sim$ satisfies *extensionality*.
Let $\vdash \alpha \leftrightarrow \beta$. Hence $\vdash \neg\alpha \leftrightarrow \neg\beta$. By definition of $\sim$ and $\circledast$ *weak extensionality* it holds that $A \sim \alpha = A \cap A \circledast \neg\alpha = A \cap A \circledast \neg\beta = A \sim \beta$.

Assume that $\circledast$ satisfies *consistency preservation, strict improvement* and *relative success*. We intend to prove that $\sim$ satisfies *success propagation*.
Let $A \sim \alpha \vdash \alpha$ and $\vdash \alpha \rightarrow \beta$. From the latter, it follows that $\vdash \neg\beta \rightarrow \neg\alpha$. By definition of $\sim$ it holds that $A \sim \alpha = A \cap A \circledast \neg\alpha$. Hence $A \circledast \neg\alpha \vdash \alpha$ and $A \vdash \alpha$. Therefore $A \vdash \beta$. On the other hand, by $\circledast$ *consistency preservation*, $\neg\alpha \notin A \circledast \neg\alpha$. From the latter and $\vdash \neg\beta \rightarrow \neg\alpha$ it follows by $\circledast$ *strict improvement* that $\neg\beta \notin A \circledast \neg\beta$. Thus by $\circledast$ *relative success* it follows that $A \circledast \neg\beta = A$. Hence $A \sim \beta = A \cap A \circledast \neg\beta = A$. Thus $A \sim \beta \vdash \beta$.

Assume that $\circledast$ satisfies *weak relative closure*. We intend to prove that $\sim$ satisfies *relative closure*.
Let $\beta \in A \cap Cn(A \sim \alpha)$. Hence $\beta \in A$. Furthermore, by definition of $\sim$, it follows that $(A \circledast \neg\alpha) \cap A \vdash \beta$. Thus by *weak relative closure* $\beta \in A \circledast \neg\alpha$. From which it follows, by definition of $\sim$, that $\beta \in A \sim \alpha$. ■

Proof of Theorem 5.9. Follows trivially by Observations 3.2, 4.4 and 2.2 and Theorem 5.7. ■

Proof of Theorem 5.10. Follows trivially by Observations 3.2, 4.4 and 4.3 and Theorem 5.8. ■

Proof of Theorem 5.12. Let $\circledast = \mathbb{R}(\sim)$ and $\sim_2 = \mathbb{C}(\mathbb{R}(\sim))$. Then:

$$A \circledast \neg\alpha = \begin{cases} (A \sim \neg\neg\alpha) \cup \{\neg\alpha\} & \text{if } A \sim \neg\neg\alpha \nvdash \alpha \\ A & \text{otherwise} \end{cases}$$

and

$$A \sim_2 \alpha = A \cap A \circledast \neg\alpha$$

By $\sim$ *extensionality* $A \sim \neg\neg\alpha = A \sim \alpha$. There are two cases to consider:
Case 1) $A \sim \neg\neg\alpha \vdash \alpha$. Then $A \circledast \neg\alpha = A$, from which it follows that $A \sim_2 \alpha = A$. On the other hand, by $\sim$ *relative success*, $A \sim \alpha = A$. Thus $A \sim_2 \alpha = A \sim \alpha$.
Case 2) $A \sim \neg\neg\alpha \nvdash \alpha$. Then $A \circledast \neg\alpha = (A \sim \neg\neg\alpha) \cup \{\neg\alpha\} = (A \sim \alpha) \cup \{\neg\alpha\}$. Hence $A \sim_2 \alpha = A \cap ((A \sim \alpha) \cup \{\neg\alpha\})$.
Let $\beta \in A \sim \alpha$. Then, by $\sim$ *inclusion*, $\beta \in A$. Hence $\beta \in A \sim_2 \alpha$. Thus $A \sim \alpha \subseteq A \sim_2 \alpha$.
Let $\beta \in A \sim_2 \alpha$. Then $\beta \in A$ and $\beta \in (A \sim \alpha) \cup \{\neg\alpha\}$. Hence $\beta \in A \sim \alpha$ or $\beta = \neg\alpha$. If $\beta = \neg\alpha$, then $\neg\alpha \in A$. From $A \nvdash \bot$ it follows that $A \nvdash \alpha$. Then, by $\sim$ *vacuity* and *inclusion*, $A \sim \alpha = A$. Therefore $\beta \in A \sim \alpha$. Hence $A \sim_2 \alpha \subseteq A \sim \alpha$. Therefore $A \sim_2 \alpha = A \sim \alpha$. ■

Proof of Theorem 5.13. Let $\sim = \mathbb{C}(\circledast)$ and $\circledast_2 = \mathbb{R}(\mathbb{C}(\circledast))$. Then:

$$A \sim \neg\alpha = A \cap A \circledast \neg\neg\alpha$$

and

$$A \circledast_2 \alpha = \begin{cases} (A \sim \neg\alpha) \cup \{\alpha\} & \text{if } A \sim \neg\alpha \nvdash \neg\alpha \\ A & \text{otherwise} \end{cases}$$

If $A \nvdash \neg\alpha$, then by $\circledast$ *vacuity* and *inclusion* $A \circledast \alpha = A \cup \{\alpha\}$ and $A \circledast \neg\neg\alpha = A \cup \{\neg\neg\alpha\}$. Thus, by $\sim$ definition, $A \sim \neg\alpha = A$. Hence $A \sim \neg\alpha \nvdash \neg\alpha$. Thus $A \circledast_2 \alpha = A \cup \{\alpha\}$. Therefore $A \circledast_2 \alpha = A \circledast \alpha$.
Assume now that $A \vdash \neg\alpha$. By $\circledast$ *relative success* $\neg\neg\alpha \in A \circledast \neg\neg\alpha$ or $A \circledast \neg\neg\alpha = A$. On the other hand, by $\circledast$ *weak extensionality* $A \cap A \circledast \neg\neg\alpha = A \cap A \circledast \alpha$. We will consider two cases:
Case 1) $A \circledast \neg\neg\alpha = A$. Hence $A \sim \neg\alpha = A$, from which it follows that $A \sim \neg\alpha \vdash \neg\alpha$. Thus $A \circledast_2 \alpha = A$.
From $A \circledast \neg\neg\alpha = A$ and $A \cap A \circledast \neg\neg\alpha = A \cap A \circledast \alpha$, it follows that $A = A \cap A \circledast \alpha$. Hence $A \subseteq A \circledast \alpha$. Since $A \vdash \neg\alpha$ and $A \nvdash \bot$ it follows, by $\circledast$ *consistency preservation*, that $\alpha \notin A \circledast \alpha$. Hence by $\circledast$ *relative success* $A \circledast \alpha = A$.
Case 2) $\neg\neg\alpha \in A \circledast \neg\neg\alpha$. Hence, by $\circledast$ *consistency preservation*, $A \circledast \neg\neg\alpha \nvdash \neg\alpha$. Thus $A \sim \neg\alpha \nvdash \neg\alpha$. Hence $A \circledast_2 \alpha = (A \sim \neg\alpha) \cup \{\alpha\} = (A \cap A \circledast \neg\neg\alpha) \cup \{\alpha\} = (A \cap A \circledast \alpha) \cup \{\alpha\}$.
Let $\beta \in A \circledast_2 \alpha$. Hence $\beta \in A \cap A \circledast \alpha$ or $\beta = \alpha$. In the former case, $\beta \in A \circledast \alpha$. Assume now that $\beta = \alpha$. If $\alpha \in A \circledast \alpha$, then $\beta \in A \circledast \alpha$.
Assume by *reductio ad absurdum* that $\alpha \notin A \circledast \alpha$. Hence, by $\circledast$ *relative success* $A \circledast \alpha = A$. Thus $A \circledast_2 \alpha = A \cup \{\alpha\}$. Hence $A \circledast_2 \alpha \vdash \bot$ and $A \circledast_2 \alpha \neq A$ (since $A \vdash \neg\alpha$ and $A \nvdash \bot$). Hence $(A \sim \neg\alpha) \cup \{\alpha\} \vdash \bot$. Therefore, by deduction, $A \sim \neg\alpha \vdash \neg\alpha$. Contradiction. Hence $A \circledast_2 \alpha \subseteq A \circledast \alpha$.
Let $\beta \in A \circledast \alpha$. By $\circledast$ *inclusion* $A \circledast \alpha \subseteq A \cup \{\alpha\}$. Hence $\beta \in A$ or $\beta = \alpha$. We will consider those two cases separately:
case 1) $\beta \in A$. Hence, by $\circledast$ *weak extensionality* it follows that $\beta \in A \cap A \circledast \neg\neg\alpha$. Therefore $\beta \in A \sim \neg\alpha$. Therefore $\beta \in A \circledast_2 \alpha$.
case 2) $\beta = \alpha$. Hence $\alpha \in A \circledast \alpha$. Assume by *reductio ad absurdum* that $A \sim \neg\alpha \vdash \neg\alpha$. Hence $A \cap A \circledast \neg\neg\alpha \vdash \neg\alpha$. From which it follows, by $\circledast$ *weak extensionality*, that $A \cap A \circledast \alpha \vdash \neg\alpha$. Therefore $A \circledast \alpha \vdash \neg\alpha$. This contradicts $\circledast$ *consistency preservation* (since $\alpha \in A \circledast \alpha$). Thus $A \sim \neg\alpha \nvdash \neg\alpha$. From which it follows by definition of $\circledast_2$ that $\alpha \in A \circledast_2 \alpha$. Thus $\beta \in A \circledast_2 \alpha$.
In both cases, $\beta \in A \circledast_2 \alpha$. Hence $A \circledast \alpha \subseteq A \circledast_2 \alpha$. Therefore $A \circledast_2 \alpha = A \circledast \alpha$. ■

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