Stability of Stationary Solutions in the Theory of Cathode Spots in Arcs in Vacuum and Ambient Gas

M. S. Benilov[1], M. D. Cunha[1], W. Hartmann[2], and N. Wenzel[2]

[1]Departamento de Física, CCCEE, Universidade da Madeira, Largo do Município, 9000 Funchal, Portugal

[2]Siemens AG, Corporate Technology, Günther-Scharowsky-Strasse 1, 91058 Erlangen, Germany

***Abstract*- Stability of stationary spots on cathodes of arcs in vacuum and ambient gas has been investigated by means of simulation of development in time of perturbations imposed over steady-state solutions. Two cases of loading conditions have been considered, namely, spots operating at fixed current (the case typical of small-scale experiments) and spots operating at fixed voltage (the case typical of high-power circuit breakers). Results are reported on spots on large copper cathodes of vacuum arcs and on spots on tungsten cathodes of high-pressure argon arcs. It is shown, in particular, that if the ballast resistance in a small-scale laboratory experiment with a high-current arc is insufficient, potential consequence may be a thermal explosion of a spot, if the arc burns in vacuum, and massive melting of the cathode surface, if the arc burns in ambient gas. This conclusion conforms to trends observed in the experiment.**

I. INTRODUCTION

2D and 3D solutions describing in a self-consistent way stationary spots on cathodes of arcs in ambient gases can be computed nowadays as a matter of routine with the use of steady-state solvers; e.g., review [1] and references therein. Recently, the same approach was applied to simulation of stationary spots on cathodes of vacuum arcs [2].

An important issue concerning solutions which describe stationary configurations and have been computed by means of steady-state solvers is their stability in some or other experimental conditions. Stability of different regimes of steady-state current transfer to cathodes of arc discharges, including regimes with spots, was investigated in the framework of the linear stability theory with the use of analytical methods [3] and the eigenvalue solver of software COMSOL Multiphysics [4]. An alternative approach to investigation of stability of stationary solutions is to perturb a stationary solution and then follow the development of the perturbed solution in time by means of a non-stationary solver. This approach, while significantly more demanding in terms of CPU time, allows one to predict the final result of development of perturbations of unstable states.

In this work, the latter approach is applied to investigation of stability of stationary solutions describing current transfer, in the first place in the spot mode, to cathodes of arcs burning in vacuum and ambient gas. Two limiting cases of loading conditions are considered, namely, spots operating at fixed current and spots operating at fixed voltage; the cases typical of, respectively, low-current arcs and high-current arcs in circuit breakers. The temporal evolution of the perturbed solution is followed until one of the four outcomes occurs: the perturbation decays, i.e., the spot returns to the original unperturbed state; the local temperature decreases down to the average temperature, i.e., the spot is extinguished; the temperature in the spot rapidly rises to very high values up to the critical temperature, which can be termed an "explosion" of the spot; the arc attachment expands and a significant part or the whole of the front surface of the cathode becomes quite hot, which can be termed "massive melting" of the cathode surface.

II. THE MODEL

The model of plasma-cathode interaction employed in this work exploits the fact that a significant power is deposited into the near-cathode space-charge sheath, which allows one to simulate the cathode and the near-cathode plasma layer independently from the arc column. In the framework of this model, distributions of temperature T and electrostatic potential φ in the cathode body are calculated by means of solving the time-dependent heat conduction equation, written with account of Joule heat generation, and the current continuity equation supplemented with Ohm's law. Boundary conditions on the cathode surface are $\kappa\, \partial T/\partial n = q(T,U)$, $\sigma\, \partial\varphi/\partial n = j(T,U)$, where q and j are densities of energy flux and electric current from the plasma to the cathode surface, which are computed in advance as functions of T, the local temperature of the cathode surface, and U the near-cathode voltage drop. κ and σ are thermal and electrical conductivities of the cathode material, which are treated as known functions of the local temperature of the cathode body. In the case of a finite cathode, the base of the cathode is assumed to be maintained at a fixed temperature T_{cool} by external cooling and electrostatic potential is set equal to zero. In the case of an infinite cathode, boundary conditions far away from the spot are $T \rightarrow T_\infty$, $\varphi \rightarrow 0$, where T_∞ is the

---

The work at Universidade da Madeira was supported by projects

PTDC/FIS-PLA/2708/2012 and PEst-OE/MAT/UI0219/2014 of FCT

978-1-4799-6752-0/14/$31.00 ©2014 IEEE

temperature of the cathode far away from the spot.

Results reported in this paper refer to infinite planar copper cathodes of vacuum arcs and rod-shaped tungsten cathodes of arcs operating in argon. The thermal and electrical conductivities of copper and tungsten were assumed the same as in [2] and [5], respectively. Functions *q(T,U)* and *j(T,U)* have been determined as described in [6] for the copper cathodes of vacuum arcs and in [7] for the tungsten cathodes of arcs burning in argon. It was set T_∞ = 1200K for the copper cathodes and T_{cool} = 300K for the tungsten cathodes.

The procedure is as follows. A stationary solution $T = T_0(\mathbf{r})$, $\varphi = \varphi_0(\mathbf{r})$, $U = U_0$ is computed by means of a steady-state solver for a given value of the spot current, $I = I_0$. Then a perturbation is imposed over this stationary solution and a nonstationary solver is invoked with this perturbed solution being the initial condition. The most of results reported in this work refer to perturbations of the distribution of the cathode temperature of the form $\beta\,[T_0(\mathbf{r}) - T_\infty]$, where β is a given parameter, with the distribution of potential being unperturbed. In other words, the initial condition for nonstationary modelling was taken in the form $T(\mathbf{r}) = T_0(\mathbf{r}) + \beta\,[T_0(\mathbf{r}) - T_\infty]$, $\varphi(\mathbf{r}) = \varphi_0(\mathbf{r})$. The non-stationary modelling is performed without perturbation of the electric control parameter: it is maintained during the whole simulation run $U = U_0$ or $I = I_0$ in the cases of voltage- and current-controlled spots, respectively. Note that the model of a spot operating at a constant voltage is appropriate for conditions of very-high current arcs, such as those in high-power circuit breakers, where many tens or even hundreds of spots operate in parallel and ignition or extinction of a spot does not affect appreciably the arc voltage. The model of a spot operating at a constant current is of interest in connection with low-current arc devices and small-scale experiments, where the arc power supply is current-controlled and there are (ideally just) one or a few cathode spots. Both steady-state and non-stationary modelling in this work was performed by means of commercial software COMSOL Multiphysics.

III. STABILITY OF VACUUM ARC SPOTS

This section is concerned with results of investigation of stability of stationary solutions describing spots on copper cathodes of vacuum arcs. As the first step, we neglect non-uniformities of the cathode surface and assume that the cathode is an infinite half-space. Detailed simulations of stationary spots on copper cathodes have been reported in [2]. Here, we only mention that the calculated temperature of a stationary spot (about 4400K), while being higher than the boiling temperature of copper, is in line with values given in other works; e.g., [8].

An example of temporal evolution of perturbations of a voltage-controlled spot is given in Fig. 1 for four levels of initial perturbations: β = ±1%, ±2%. The maximum temperature in the cathode body, T_{max}, is shown on different scales in figures 1a and 1b. Also shown in Fig. 1b is the temperature of the cathode surface at the center of the spot, T_c. Note that in the cases where the initial perturbations of cathode temperature are negative (β<0), the highest temperature in the cathode body at all times is attained on the cathode surface at the center of the spot, so $T_{max} = T_c$ and the corresponding lines in Fig. 1b are indistinguishable. The spot current is shown in Fig. 1c. The horizontal lines in figures 1a-1c represent the steady-state values.

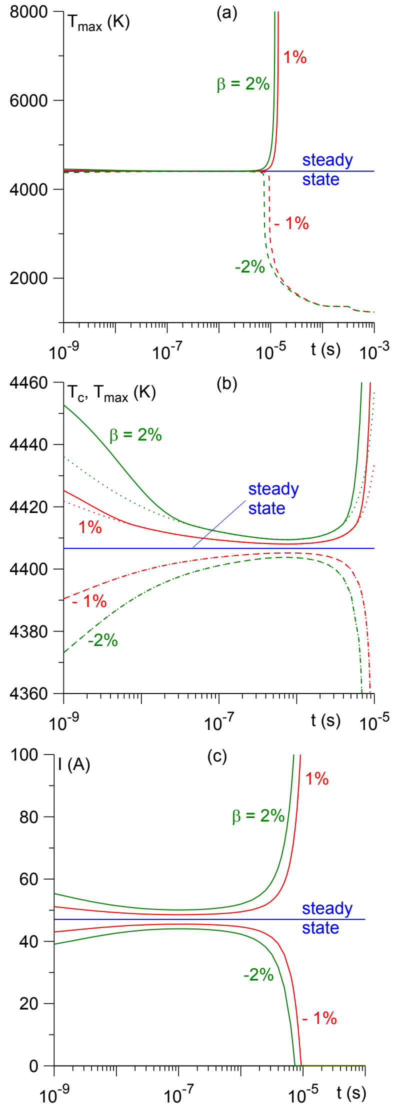

Fig. 1. Development of perturbations of a voltage-controlled vacuum spot. U = 20 V. (a) Maximum temperature in the cathode body, T_{max}. (b) T_{max} (solid, dashed) and T_c the temperature of the cathode surface at the center of the spot (dotted). (c): Spot current.

At the first phase, the perturbations decrease on the time scale of nanoseconds, so T_{max} and I approach the steady-state values. Note that in the case of positive perturbations (β>0) the cathode surface at this phase is cooled down faster than the interior of the cathode, so the temperature maximum is shifted inside the cathode and $T_{max} > T_c$. During some microseconds, T_{max} and I remain close to the stationary values, and the spot radius as well (the corresponding figure is skipped for brevity). At around 10 µs the perturbations start growing once again and rapidly enter the nonlinear phase. In the case of positive perturbations, the temperature maximum is shifted from the surface into the cathode volume and in the course of around one microsecond or less T_{max} and I increase up to extremely high values. One can say that a thermal explosion occurs. In the case of negative perturbations, the spot cools down to temperatures below 2000 K in the course of a few microseconds, after which it continues cooling down at a slower rate and at t of the order of 100 µs T_{max} is virtually indistinguishable from T_∞. One can say that the spot is destroyed by thermal conduction.

Thus, stationary solutions describing steady-state spots are unstable if the near-cathode voltage is not affected by processes in a spot. There are two scenarios of the nonlinear development of perturbations: thermal explosion for "positive" perturbations and destruction of the spot by thermal conduction for "negative" perturbations.

The region of very high temperatures underneath the cathode surface that is formed in the course of development of positive perturbations is a consequence of Joule heat generation in the cathode body. In order to illustrate this relation, the development of positive perturbations of a voltage-controlled vacuum spot computed without account of Joule heating was calculated. It was found that the spot remains unstable, however the outcome of the development of perturbations changes: the increase of the maximum temperature (which occurs on the cathode surface) during the nonlinear phase is quite modest, while the spot radius increases to extremely high values. One can say that positive perturbations result in massive melting of the cathode surface in the case where there is no Joule heat generation in the cathode body. Note that negative perturbations result in destruction of the spot by thermal conduction as before.

Calculations performed under conditions for which the spot current is maintained fixed have shown that stationary spots are stable under such conditions. This conclusion conforms to results of the 0D analysis performed in [8], where the spot is described only on integral level, according to which a steady-state solution can be reached in the course of evolution of a spot at a fixed current.

IV. STABILITY OF CURRENT TRANSFER TO CATHODES OF ARCS IN AMBIENT GAS

As an example, let us consider conditions of experiments with high-current arcs [9] typical for, e.g., welding devices or plasma torches. In this case, the cathode is made of tungsten and the arc operates in atmospheric-pressure argon plasma; the shape of the cathode is a cylinder with hemispherical tip of the radius of 1 mm and height of 12 mm.

Voltage-controlled steady states are unstable, similarly to voltage-controlled cathode spots of vacuum arcs. As an example, the temporal evolution of a perturbation of a voltage-controlled steady state with U = 30 V (and I = 6.0 A) is shown in Fig. 2a for the initial perturbation β = 1%. d in figures 2b and 2c is the distance from the center of the front surface of the cathode measured along the generatrix, so the range $0 \leq d \leq 1.57$ mm corresponds to the front (spherical) surface of the cathode while the range $d > 1.57$ mm corresponds to the lateral (cylindrical) surface.

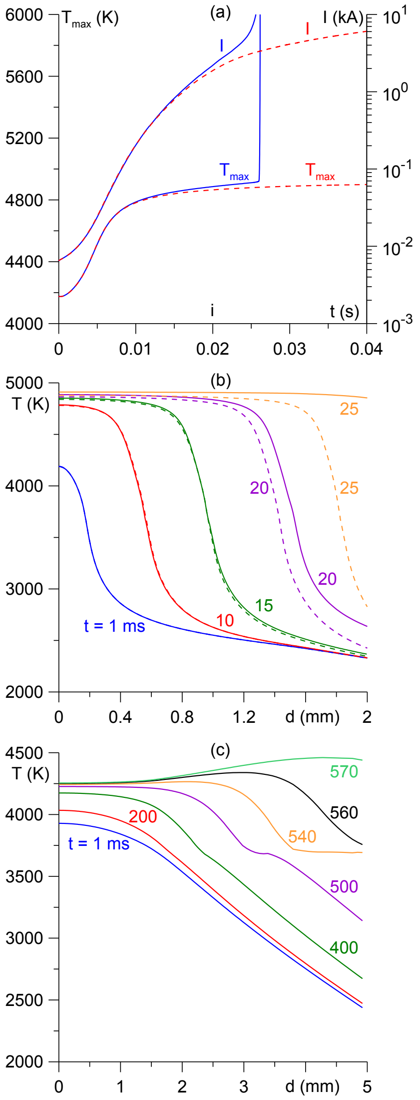

Fig. 2. Development of perturbations of voltage-controlled current transfer to cathode of an argon arc. β=1%. Dashed: simulations without account of Joule heating. (a) Maximum temperature in the cathode body, T_{max}, and spot current. (b) and (c) Distributions of the temperature along the cathode surface. (a) and (b) U = 30 V (I = 6.0 A). (c) U = 10.2 V (I = 100 A).

There is no initial decrease of perturbation on the nanosecond scale, in contrast to the case of cathode spots of vacuum arcs, and perturbations develop on the scale of milliseconds. During the first 10 ms the area on the cathode surface heated to temperatures above 4000 K expands from $d \lesssim 0.2$ mm to $d \lesssim 0.6$ mm, the arc current increases from 6 A to approximately 200 A, and T_{max} increases from 4200 K to around 4800 K. In the interval 10 ms $\lesssim t \lesssim$ 25 ms T_{max} does not change much, however the area of the hot region and I continue to grow steadily. At around 25 ms the temperature maximum is shifted from the surface into the cathode volume and both T_{max} and I start growing in an explosive-like way, however by that time the whole region $d \lesssim 3$ mm has attained a temperature exceeding 4000 K and the arc current exceeds 1 kA. In other words, the whole front surface of the cathode is melted before the explosion could occur. The Joule heating does not affect significantly the massive melting; it would have occurred also in the absence of Joule heating.

As seen from the curve labeled 1 ms in Fig. 2b, the above-considered steady state is associated with a small (of radius around 0.2 mm) spot at the center of the front surface of the cathode, which is typical for the low-current regime. Evolution of perturbations of voltage-controlled steady states with high currents is illustrated by Fig. 2c, which refers to the state with U = 10.2 V and I = 100 A. There is no spot in this steady state: the whole front surface of the cathode collects current. Again a massive heating occurs, however on a longer time scale. Note that the maximum of temperature on the lateral surface which is seen for $t \geq$ 540 ms is due to Joule heating in the cathode body, as discussed in [5], and may be viewed as a precursor of thermal explosion which would have happened a bit later.

Calculations performed for negative perturbations of voltage-controlled steady states show that such perturbations result in destruction of steady states by thermal conduction as in the case of cathode spots of vacuum arcs, although this happens slower.

Current-controlled steady states of current transfer are stable except intermediate states in the range of hysteresis [5]. These states are unstable: positive thermal perturbations result in a transition to a state belonging to the regime with a diffuse temperature distribution (and a lower T_{max}), and negative perturbations result in a transition to a state belonging to the regime with a hot spot (and a higher T_{max}). It is interesting to note that thermal perturbations change sign in the course of their development in this case.

V. CONCLUSIONS

Stationary solutions describing steady-state spots on cathodes of vacuum arcs are unstable if the near-cathode voltage is not affected appreciably by ignition or extinction of an individual spot; the case typical of high-power circuit breakers where many tens or even hundreds of spots operate in parallel. Two scenarios of development of the nonlinear phase of instability have been found: thermal explosion for "positive" thermal perturbations and destruction of the spot by thermal conduction for "negative" perturbations. The fact that thermal explosion occurs even in the case of planar cathode, which is the one treated in this work, is remarkable, although in this case the time of its beginning is rather long, of the order of 10 μs. Stationary spots are stable if the spot current is not affected by processes in the spot; the case typical for small-scale laboratory experiments.

Stability of stationary regimes of axially symmetric current transfer under conditions of high-current arcs in ambient gas is similar but is not quite the same: the instability of voltage-controlled states develops significantly slower and in the case of positive perturbations results in massive melting of the cathode surface rather than in thermal explosion; current-controlled steady states are stable except intermediate states in the range of hysteresis.

It follows that if the ballast resistance in a small-scale laboratory experiments with a high-current arc is insufficient, potential consequence may be thermal explosion of the spot, if the arc burns in vacuum, and massive melting of the cathode surface, if the arc burns in an ambient gas. This conclusion conforms to trends observed in the experiment.

REFERENCES

[1] M. S. Benilov, "Understanding and modelling plasma-electrode interaction in high-pressure arc discharges: a review," J. Phys. D: Appl. Phys. 41, 144001 (2008).

[2] M. S. Benilov, M. D. Cunha, W. Hartmann, S. Kosse, A. Lawall, and N. Wenzel, "Space-resolved modelling of stationary spots on copper vacuum arc cathodes and on composite CuCr cathodes with large grains," IEEE Trans. Plasma Sci. 41, 1950 (2013).

[3] M. S. Benilov, "Stability of direct current transfer to thermionic cathodes: I. Analytical theory," J. Phys. D: Appl. Phys. 40, 1376 (2007).

[4] M. S. Benilov and M. J. Faria, "Stability of direct current transfer to thermionic cathodes: II. Numerical simulation," J. Phys. D: Appl. Phys. 40, 5083 (2007).

[5] M. S. Benilov and M. D. Cunha, "Joule heat generation in thermionic cathodes of high-pressure arc discharges," J. Appl. Phys. 113, 063301 (2013).

[6] N. A. Almeida, M. S. Benilov, L. G. Benilova, W. Hartmann, and N. Wenzel, "Near-cathode plasma layer on CuCr contacts of vacuum arcs," IEEE Trans. Plasma Sci. 41, 1938 (2013).

[7] M. S. Benilov, M. D. Cunha, and G. V. Naidis, " Modelling current transfer to cathodes in metal halide plasmas," Plasma Sources Sci. Technol. 14, 517 (2005).

[8] I. I. Beilis, "Cathode Spot Development on a Bulk Cathode in a Vacuum Arc," IEEE Trans. Plasma Sci. 41, 1979 (2013).

[9] N. K. Mitrofanov and S. M. Shkol'nik, "Two Forms of Attachment of an Atmospheric-Pressure Direct-Current Arc in Argon to a Thermionic Cathode," Tech. Phys. 52, 711 (2007).

E-mail of the author(s): benilov@uma.pt