

DM

On the Iteration of Katsuno and Mendelzon Update

DISSERTAÇÃO DE MESTRADO

Sara Xavier Reis Gonçalves Rodrigues

MESTRADO EM MATEMÁTICA



UNIVERSIDADE da MADEIRA

A Nossa Universidade

www.uma.pt

novembro | 2015

On the Iteration of Katsuno and Mendelzon Update

DISSERTAÇÃO DE MESTRADO

Sara Xavier Reis Gonçalves Rodrigues

MESTRADO EM MATEMÁTICA

ORIENTADOR

Eduardo Leopoldo Fermé

“If I had a world of my own, everything would be nonsense. Nothing would be what it is, because everything would be what it isn’t. And contrary wise, what is, it wouldn’t be. And what it wouldn’t be, it would. You see?”,
Lewis Carroll, Alice’s Adventures in Wonderland

Abstract

In this dissertation we present a model for iteration of Katsuno and Mendelzon's Update, inspired in the developments for iteration in AGM belief revision. We adapt Darwiche and Pearl's postulates of iterated belief revision to update (as well as the independence postulate proposed in [BM06, JT07]) and show two families of such operators, based in natural [Bou96] and lexicographic revision [Nay94a, NPP03]. In all cases, we provide a possible worlds semantics of the models.

Keywords:

Belief Change, Belief update, Katsuno and Mendelzon Update, Iteration, Possible World Semantic

Resumo

Nesta dissertação é apresentado um modelo para a iteração de Update de acordo com a definição de Katsuno e Mendelzon, inspirado nos desenvolvimentos da iteração na revisão de crenças AGM. Serão adaptados para iteração de update os postulados para iteração de revisão de crenças de Darwiche e Pearl (bem como o postulado de Independência proposto em [BM06, JT07]) e serão mostradas duas famílias de tais operadores, baseadas na revisão Natural [Bou96] e Lexicográfica [Nay94a, NPP03]. Em todos os casos será apresentada uma semântica de mundos possíveis.

Palavras-chave:

Revisão de Crenças, Update de Crenças, Modelo de Update de Katsuno e Mendelzon, Iteração, Semântica de mundos possíveis

Acknowledgements

To all the teachers who accompanied me throughout my academic life and that were able to make me find love for mathematics and thirst for knowledge.

In particular to Doctor Eduardo Fermé, who introduced me to logic and that made me wish that I would, one day, finish this dissertation, more and above all because of the personal satisfaction that it represents.

To my colleagues and friends, whether from academic or professional life, which followed more closely this task and managed to realize what it means to me.

To those, who despite no longer present, live in my heart and follow my soul every day, giving me their hand and taking me in their arms whenever necessary.

To my mother, my brother and my sister for reading the early stages of my work, even if not understanding it.

To my mother-in-law and especially and most of all to my husband, Valter, who early on took this project as if his own and encouraged and supported me in all its stages. Thank you for making me love you every day!

Contents

List of Figures	xv
1 Introduction	1
1.1 Historic Background	1
2 The “AGM” Theory	5
2.1 Formal preliminaries	5
2.2 The AGM Model	5
2.3 The modified Katsuno and Mendelzon postulates	7
2.4 Revision Models	9
2.4.1 Dalal’s Revision	9
2.4.2 Borgida’s Revision	10
2.4.3 Winslett’s Revision	11
2.4.4 Satoh’s Revision	11
2.4.5 Weber’s Revision	12
3 Iterated belief revision	13
3.1 Definition	13
3.2 Iterable Operators	15
3.2.1 Independence Postulate	15
3.2.2 Conservative and Lexicographic Revision	16
4 The Katsuno and Mendelzon Update	19
4.1 Definition	19
4.2 Update Operators	21
4.2.1 PMA	21
4.2.2 FORBUS	21
4.2.3 MCD	22
5 Iteration for KM-update	23
5.1 Definition	23
5.2 Two families of KM-update	27
6 Conclusion	29
Bibliography	32

List of Figures

2.1	The six types of belief change.	6
2.2	Belief Revision	9
3.1	AGM doesn't provide insight to iteration.	13
3.2	An example of a belief revision function satisfying (CR1)-(CR4).	15
3.3	An example of an admissible revision function.	16
3.4	An example of Natural Revision (left) and Lexicographic revision (right).	17
4.1	An example of an update operator	20
5.1	An example of an update operator satisfying (CU1)-(CU4) and (C-Ind) of $[[\varphi \blacklozenge \alpha]]$	26
6.1	The total preorders associated to Example 1.	30

Chapter 1

Introduction

1.1 Historic Background

Belief change is the process through which an agent should revise its beliefs upon the arising of new information and it is present in our day to day lives. Even though we are not aware, changes occur all around us, second after second for every second that passes by. These changes lead to a constant renewal of our beliefs. We end up accepting new beliefs, revising old ones and even eliminating some. In humans these changes occur naturally, they are a part of our nature and of the way our brain functions.

It is only natural that with the development of computer science, machines are expected to react the same way whenever there is a change in the environment or even when the environment changes, giving us consistent outcomes, just as a human brain would do. However, the formalization of this problem is not simple, due to the fact that there isn't an unique way to perform these changes.

The belief change study, when referred to in its wide sense, has been a subject of interest for almost as long as man itself exists. In the modern age, the most various works primarily originated in two large research traditions: philosophy and computer science. Numerous contributions in the field of artificial intelligence were given by researchers as Alexander Borgida, Mukesh Dalal, Stuart Shapiro, Ken Satoh and Marianne Winslett [Bor85, Dal88a, MS88, Sat88, Win88]. In the philosophical field, the great breakthrough occurred with the works of Isaac Levi [Lev77], William Harper [Har77] and Georg Henrik von Wright [vW71]. Through the study of the mechanisms by which scientific theories are developed, philosophers proposed criteria of rationality for revisions of probability assignments, providing much of the basic formal framework of belief change.

It was, however, the work of Carlos Alchourrón, David Makinson and Peter Gärdenfors that promoted the development of the belief change study area primarily in the philosophical field. The first two researchers were studying the changes in legal codes, analysing the logical structure of the derogation procedure of a norm contained in a legal code [AM81], that is, the process of eliminating a norm from the legal code (derogation is the term used in the deontic context and is similar to contraction when talking in theory change). They wanted to find the general principles that any derogation

should satisfy, and define a family of all the possible ones. Basically, given a code A , a partial order between the norms of A should be created, inducing an order on the set of parts of A . The maximal sets of A that did not involve the norm to be removed were called “remainders”. An year later [AM82] they extended the problem, studying not a set of norms but an arbitrary set of formulae. The problem now was how to eliminate one of the formulae or one of the consequences of the set. They analyzed two different ways to contract a theory by means of remainder sets: Maxichoise and Full Meet.

At almost the same time Peter Gärdenfors, a researcher from the same study area as Alchourrón and Makinson, was interested in the connections between belief change and conditional sentences [Gär78]. He was looking for a model for explanations and believed that these could be expressed as different types of conditional sentences. The influence he received from the philosophers Levi and Harper led him to make an exhaustive study of epistemic conditionals. Gärdenfors was looking for a semantic for the epistemic conditionals that had to be based on belief states and belief changes. He defined a set of postulates that change functions should satisfy [Gär82].

In 1985, considering the closeness of their ideas, these three researchers united forces and wrote a paper called “On the Logic of Theory Change: Partial Meet Contraction and Revision Function” [AGM85] where they conceived the famous “AGM” model: a new and formal framework that 30 years later continues to be the subject of significant study and development and remains the core of the belief change theory, acquiring the status of a standard model of belief change.

Given its properties, the AGM model inspired many researchers to propose extensions and generalizations as well as applications and connections with other fields (for an overview of these proposals see [FH11]). One of these extensions was iterated change: a drawback of the AGM definition of revision is that the conditions for the iteration of the process are very weak, and this is caused by the lack of expressive power of belief sets. In order to ensure good properties for the iteration of the revision process, one needs a more complex structure. So shifting from belief sets to epistemic states was proposed by Darwiche and Pearl in [DP96]. In this framework, it is possible to define interesting iterated revision operators. Although there exists several ways to introduce iteration in the AGM operators (for an overview see [Pep14]), our dissertation will be based on that of Darwiche and Pearl.

In 1992, Katsuno and Mendelzon (KM) presented a type of operator of change that they called update [KM92]. Whereas AGM operators are suited to capture changes that reflect evolving knowledge about a static situation, the KM-update operators are intended to represent changes in beliefs that result from changes in the objects of belief. The difference was pointed out for the first time by Keller and Winslett [KW85] (in the context of relational databases) and is captured in the following example [Win88]:

Initially the agent knows that there is either a book on the table (p) or a magazine on the table (q), but not both.

Case 1: The agent is told that there is a book on the table. She concludes that

there is no magazine on the table. This is revision.

Case 2: The agent is told that subsequently a book has been put on the table. In this case she should not conclude that there is no magazine on the table. This is update.

In this dissertation we propose to analyze iteration in the context of update, inspired in AGM revision. An important difference between AGM revision and KM-update is that belief revision is a local operation in the sense that a revision function is defined just for the current belief set and the language. KM-update, on the other hand, is a global operator defined for the set of all the possible belief sets and the language. Since update is defined as a global operator, an operator \diamond is defined for all the possible belief sets φ . Consequently, in a first view, iteration is not a problem since $(\varphi \diamond \alpha) \diamond \beta$ is well defined. However, what happens if we want to make changes in our preferences as a consequence of updating by α ? In that case we can define a new operator \diamond_α to reflect these changes. This implies that, $(\varphi \diamond \alpha) \diamond \beta$ and $\varphi \diamond_\alpha \beta$ are different operations (in the conclusion section we will show this difference).

This dissertation will first describe the AGM Model and the modified postulates of AGM, as defined by Katsuno and Mendelzon [KM91]. Some revision models will be characterized. On Chapter 3 iterated belief revision will be explained using the work of Darwiche and Pearl [DP96] as well as the Independence Postulate proposed by Booth and Meyer and by Jin and Thielscher in [BM06, JT07]). Two different families of operators for iterated revision (based in Natural [Bou96] and Lexicographic revision [Nay94a, NPP03]) will be presented.

On Chapter 4 one can find the definition of the KM-update, as well as some update operators, common in the literature of this theme. Chapter 5 will present the goal of this work, that is, the adaptation of the Darwiche and Pearl's postulates of iterated belief revision to iterated update.

The dissertation will be concluded on Chapter 6, which is dedicated to some discussion about its content.

Chapter 2

The “AGM” Theory

2.1 Formal preliminaries

We denote by \mathcal{L} the set of formulas of a propositional language built over a finite set of propositional variables \mathcal{P} . The elements of \mathcal{L} are denoted by lower case Greek letters α, β, \dots (possibly with subscripts). The set of valuation functions from the set of propositional variables into the boolean set $\{0, 1\}$ (*false, true*) is denoted \mathcal{V} .

We write $\omega \models \alpha$ when a valuation $\omega \in \mathcal{V}$ satisfies a formula α , *i.e.* when ω is a model of α . The set of models of a formula α is denoted by $\llbracket \alpha \rrbracket$. If M is a set of models we denote by α_M a formula such that $\llbracket \alpha_M \rrbracket = M$. When the size of M is small we often omit the braces, by writing, e.g., $\alpha_{\omega, \omega'}$ instead of $\alpha_{\{\omega, \omega'\}}$. The set of consistent formulas will be denoted \mathcal{L}^* .

If \leq is a total pre-order (a total and transitive relation), then \simeq is a notation for the associated equivalence relation ($a \simeq b$ iff $a \leq b$ and $b \leq a$), and $<$ is the notation for the associated strict order ($a < b$ iff $a \leq b$ and $b \not\leq a$).

2.2 The AGM Model

In the classical “AGM” theory, as presented by Carlos Alchourrón, David Makinson and Peter Gärdenfors [AGM85], beliefs are represented by belief sets K (logically closed sets of sentences) in a language \mathcal{L} and a change consists on adding or removing a sentence to/from a belief set in order to obtain a new belief set.

Provided that the belief set is consistent, the epistemic agent can have exactly three epistemic attitudes towards a sentence α , each defined from the belief set:

1. $\alpha \in K$: belief in α ;
2. $\neg\alpha \in K$: disbelief or rejection of α ;
3. $\alpha \notin K$ and $\neg\alpha \notin K$: suspension of belief or α unsettled.

Three operations were then defined:

- Expansion: a sentence is added to the belief set and nothing is removed (represented as $+\alpha$ or $+\neg\alpha$);
- Contraction: a sentence is removed from the belief set and nothing is added (represented as $-\alpha$ or $-\neg\alpha$);
- Revision: a new sentence is added to the belief set and at the same time other sentences are removed if necessary to ensure the consistency of the revised set (represented as $*\alpha$ or $*\neg\alpha$).

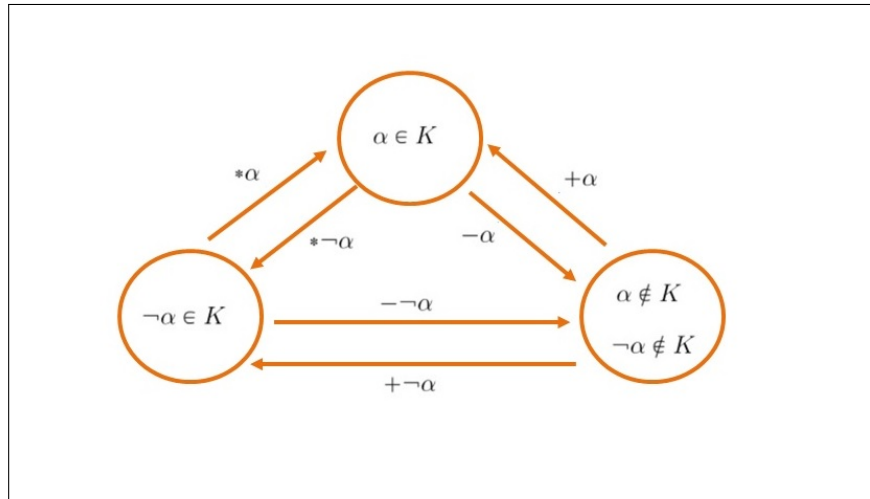


Figure 2.1: The six types of belief change.

When talking of the different operations defined by the AGM model, one can see that they are related: in the revision process new beliefs are incorporated (just as it happens in expansion) but in order to preserve consistency some sentences have, sometimes, to be eliminated (just as it happens with contraction).

The process of revising a belief is modeled as a function $*$ from $\mathcal{K} \times \mathcal{L}$ to \mathcal{K} and $K * \alpha$ represents the revision of the belief set K (deductively closed set of formulas) by the sentence α . In order to understand and define this process, eight postulates, known as the AGM postulates, were described by the referred article authors:

- (AGM1) $K * \alpha$ is a belief set
- (AGM2) $K * \alpha \vdash \alpha$
- (AGM3) $K * \alpha \subseteq K + \alpha$ (where $+$ represents the expansion operator)
- (AGM4) If $K \not\vdash \neg\alpha$ then $K + \alpha \subseteq K * \alpha$ (i.e., $K + \alpha = K * \alpha$)
- (AGM5) If $\not\vdash \neg\alpha$ then $K * \alpha \neq K_{\perp}$
- (AGM6) If $\vdash \alpha \leftrightarrow \beta$ then $K * \alpha = K * \beta$
- (AGM7) $K * (\alpha \wedge \beta) \subseteq (K * \alpha) + \beta$
- (AGM8) If $K * \alpha \not\vdash \neg\beta$ then $(K * \alpha) + \beta \subseteq K * (\alpha \wedge \beta)$

K is a belief set, α and β are logical formulas belonging to the language \mathcal{L} and $*$ is a binary operator for revision that takes the current beliefs and the new information and produces a new belief set that represents the result of the revision.

The first AGM postulate states that the result of revising the belief set K with α is still a belief set. This postulate is known as closure.

Postulate (AGM2), known as success, ensures that the new sentence is incorporated in the revision, that is, $\alpha \in K * \alpha$.

Success, in combination with the third postulate (known as inclusion), guarantees that the revised belief set consists in the logical consequence of the new belief and a subset of sentences of K that do not contradict the new belief.

The fourth postulate, vacuity, essentially says that if the new belief does not contradict any of the sentences in K , there is no reason to remove any of them.

With the consistency fifth postulate, one can be sure that unless the new belief is itself inconsistent, the result of the revision is consistent. Let us note that $K * \alpha$ is consistent even if α is consistent. In this matter, revision differs from the process of update, which we will refer to in Chapter 4.

The revision operation must be independent of the syntactic representation of the sentences, that is, logically equivalent sentences must yield the same result. This is what is stated by postulate (AGM6) or the extensionality criteria.

These six postulates form the basic AGM postulates for revision. The two following ones, seventh and eight postulates are called superexpansion and subexpansion and are presented in the terms of revision by a conjunction, but they are equivalent to a set of postulates of revision by a disjunction as demonstrated by Peter Gärdenfors in [Gär88].

According to the AGM authors, if K is to be changed minimally so as to include two sentences α and β , such a change should be possible by first revising K with respect to α and then expanding $K * \alpha$ by β , provided that β does not contradict the beliefs in $K * \alpha$.

2.3 The modified Katsuno and Mendelzon postulates

A few years after the publication of the “AGM” model, Hirofumi Katsuno and Alberto O. Mendelzon rephrased its postulates using a propositional logic setting [KM91]. In other words, they used a finite knowledge base instead of a knowledge set.

They represented a belief set by a sentence φ in a propositional language \mathcal{L} , where any sentence that is entailed by φ is part of the belief set. Evidence is also represented using a sentence μ in \mathcal{L} and the result of revising φ with μ is a sentence denoted by $\varphi \circ \mu$, where \circ is called a belief revision operator. The sentence $\varphi \circ \mu$ also belongs to the language \mathcal{L} .

By representing any knowledge set K by a propositional formula φ such that $K = \phi | \varphi \vdash \phi$, a direct correspondence between $K * \mu$ and $\varphi \circ \mu$ was established:

Lemma 1 [KM91] *Let $*$ be a revision operator on knowledge sets and \circ its corresponding operator on knowledge bases. Then $*$ satisfies (AGM1) - (AGM6) if and only if \circ satisfies conditions (R1) - (R4) below:*

- (R1) $\varphi \circ \mu$ implies μ
- (R2) If $\varphi \wedge \mu$ is satisfiable, then $\varphi \circ \mu \equiv \varphi \wedge \mu$
- (R3) If μ is satisfiable, then $\varphi \circ \mu$ is also satisfiable
- (R4) If $\varphi_1 \equiv \varphi_2$ and $\mu_1 \equiv \mu_2$, then $\varphi_1 \circ \mu_1 \equiv \varphi_2 \circ \mu_2$

This lemma allows us to understand the first six AGM postulates, while the following lemma redefines the remaining two:

Lemma 2 [KM91] *(AGM7) and (AGM8) are equivalent to (R5) and (R6) respectively in the sense of the previous Lemma:*

- (R5) $(\varphi \circ \mu) \wedge \phi$ implies $\varphi \circ (\mu \wedge \phi)$
- (R6) If $(\varphi \circ \mu) \wedge \phi$ is satisfiable, then $\varphi \circ (\mu \wedge \phi)$ implies $(\varphi \circ \mu) \wedge \phi$

Along with this definition Katsuno and Mendelzon provided a representation theorem that shows an equivalence between the postulates and a revision mechanism based on total pre-orders. These are defined as:

Definition 1 [KM91] *Let W be the set of all worlds (or interpretations) of a propositional language \mathcal{L} . A function that maps each sentence φ in \mathcal{L} to a total pre-order \leq_φ on worlds W is called a faithful assignment if and only if:*

- (1) $\omega_1, \omega_2 \models \varphi$ only if $\omega_1 =_\varphi \omega_2$
 - (2) $\omega_1 \models \varphi$ and $\omega_2 \not\models \varphi$ only if $\omega_1 <_\varphi \omega_2$
- and
- (3) $\varphi \equiv \phi$ only if $\leq_\varphi = \leq_\phi$

Here $\omega_1 <_\varphi \omega_2$ is defined as $\omega_1 \leq_\varphi \omega_2$ and $\omega_2 \not\leq_\varphi \omega_1$ and $\omega_1 =_\varphi \omega_2$ is defined as $\omega_1 \leq_\varphi \omega_2$ and $\omega_2 \leq_\varphi \omega_1$.

Katsuno and Mendelzon’s representation theorem shows that a revision operator is equivalent to a faithful assignment where the result of a revision $\varphi \circ \mu$ is determined by μ and the total pre-order assigned to φ :

Theorem 1 [KM91] *A revision operator \circ satisfies postulates (R1) - (R6) precisely when there exists a faithful assignment that maps each sentence φ into a total pre-order \leq_{φ} such that*

$$\text{Mods}(\varphi \circ \mu) = \min(\text{Mods}(\mu), \leq_{\varphi}).$$

Where $\text{Mods}(\mu)$ is the set of all worlds satisfying μ and $\min(\text{Mods}(\mu), \leq_{\varphi})$ contains all worlds that are minimal in $\text{Mods}(\mu)$ according to the total pre-order \leq_{φ} , i.e. all the worlds that include μ and are closer to φ .

Figure 2.2 provides a graphical representation of the possible worlds approach¹.

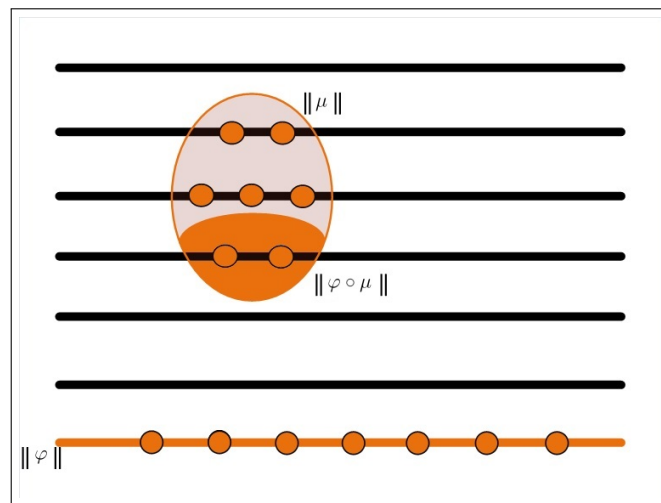


Figure 2.2: Belief Revision

2.4 Revision Models

The AGM paper authors proposed eight postulates that should be satisfied by any reasonable revision function. Those postulates were formulated in a very general setting, and were later transformed by Katsuno and Mendelzon, using a propositional logic setting.

Throughout the years, several investigators presented definitions of concrete revision operators. Some are able to satisfy the AGM postulates while others fail for some reason. Below are some known revision operators proposals, that can be found in the literature (for a summary on these please refer to [KM89]).

2.4.1 Dalal's Revision

When talking of revision models, Dalal's revision operator is one of the most important ones. It uses the number of propositional letters on which two interpretations differ as a

¹This graphical notation is due to Konieczny and Pino Pérez.

measure of “distance” between them, thus inducing a new interpretation ordering.

Let us take two interpretations I and J and define the distance between them, $dist(I, J)$, as the total number of propositional letters on which I and J differ. This distance, also known as the Hamming distance is defined as:

$$dist(I, J) = |I \Delta J|, \text{ where the } |\cdot| \text{ operator is set-cardinality.}$$

The distance between a formula $Mod(\varphi)$ and an interpretation I is defined as:

$$dist(Mod(\varphi), I) = \min_{J \in Mod(\varphi)} dist(J, I)$$

A persistent assignment of a total pre-order \leq_{φ} can be determined as $I \leq_{\varphi} J$ if and only if:

$$dist(Mod(\varphi), I) \leq dist(Mod(\varphi), J)$$

Finally, Dalal’s revision operator, \circ_D is as follows:

$$Mod(\varphi \circ_D \mu) = Min(Mod(\mu), \leq_{\varphi})$$

This operator satisfies (AGM1)-(AGM8) postulates, thus satisfying Katsuno and Mendelzon’s postulates as well.

2.4.2 Borgida’s Revision

The revision operator proposed by Borgida [Bor85] and later extended by Dalal [Dal88b] orders interpretations according to the set-inclusion of symmetric set-differences. An interpretation I can be thought as a set that only contains the propositional variables that hold on I and the symmetric set-difference $I \Delta J$ of two interpretations I and J is the set that contains all the propositional variables whose values differ in I and in J .

Therefore, given a formula μ and an interpretation I , the set of differences between them can be defined as:

$$diff(I, \mu) = I \Delta J | J \in Mod(\mu)$$

Borgida’s revision operator, \circ_B is defined as follows:

If $\varphi \wedge \mu$ is consistent, then $\varphi \circ_B \mu = \varphi \wedge \mu$;

Otherwise, J is a model of $\varphi \circ_B \mu$ if there is a model I of φ , such that $I \Delta J \in Min(diff(I, \mu))$

This revision operator is known to satisfy (R1) - (R5) but not (R6).

2.4.3 Winslett's Revision

In 1988 Winslett [Win88] proposed a revision operator defined for the first order calculus case and called possible models approach (PMA). Her starting point was the possible worlds approach (PWA) proposed in 1987 by Ginsberg and Smith [GS88a, GS88b], and that can be summed up as:

“To incorporate a set S of formulas into a theory T , take the maximal subset T' of T that is consistent with S , and add S to T' ”, or, in other words: “as little as possible in the description of the world changes when an action is performed”.

PMA is actually very similar to PWA: when using PMA it's the models of T (instead of the formulas of T) that are to be changed as little as possible to make S true. Let us define the function $Incorporate(S, M)$, the set of models produced by incorporating S into M :

Let M be a model of T and let S be a set of formulas. $Incorporate(S, M)$ is the set of all models M' such that:

1. S and the protected formulas of T are true in M' ;
2. No other model satisfying 1. differs from M on fewer atoms, where fewer is defined by set inclusion.

The possible states of the world resulting from applying an action with postconditions S are given by:

$$\bigcup_{M \in Models(T)} Incorporate(S, M)$$

Let us restrict this operator, \circ_{pma} , to the propositional case. If the old knowledge base is inconsistent with the new knowledge, \circ_{pma} coincides with Borgida's operator \circ_B . However, even if the new knowledge is consistent with the knowledge base, Winslett defines \circ_{pma} in the same way. Thus, \circ_{pma} violates condition (R2).

2.4.4 Satoh's Revision

The revision operator proposed by Satoh [Sat88] is defined in first order logic and if applied to the propositional logic case corresponds to a global version of Borgida's revision operator.

Given two formulas φ and μ , the set of differences between them is defined as:

$$diff(\varphi, \mu) = \bigcup_{I \in Mod(\varphi)} diff(I, \mu)$$

Satoh defines an interpretation J to be a model of $\varphi \circ_S \mu$ if there exists a model I of φ such that $I \Delta J$ is a minimal element of $\text{diff}(\varphi, \mu)$.

Satoh’s revision operator, \circ_S satisfies (R1) - (R5) but not (R6).

2.4.5 Weber’s Revision

Weber’s revision operator [Web86] also concentrates on sets of propositional letters on which a model of φ and a model of μ differ and can be defined as:

If φ and μ are both satisfiable, then $\varphi \circ_W \mu$ is defined by $\text{res}_\Omega \wedge \mu$, where Ω is the union of the minimal sets of $\text{diff}(\varphi, \mu)$.

If either φ or μ is unsatisfiable, then $\varphi \circ_W \mu$ is defined as φ .

If φ and μ are both consistent, Weber’s revision operator satisfies (R1) - (R4) but not (R5) nor (R6).

Chapter 3

Iterated belief revision

3.1 Definition

Although the AGM paper was of most importance to the development of the area of belief revision, their postulates were not sufficient to ensure the rational preservation of conditional beliefs, that is, the sequential revision of beliefs in response to a string of observations. The AGM postulates are not able to capture the dynamics of the structure used to encode one-step revision policies and are, therefore, too weak to properly regulate iterated belief revision.

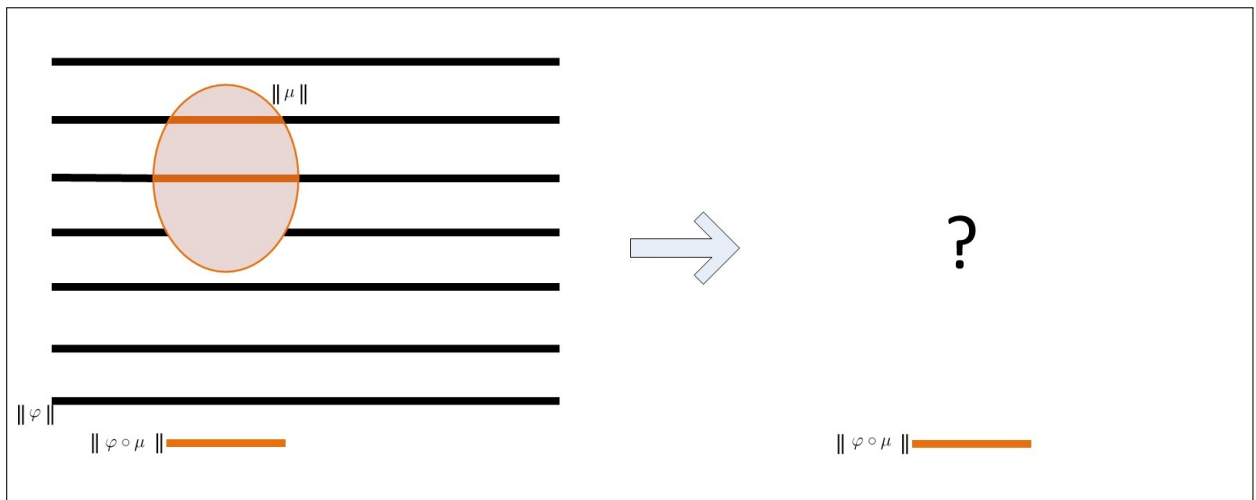


Figure 3.1: AGM doesn't provide insight to iteration.

With this in mind Adnan Darwiche and Judea Pearl wrote an article [DP96] in which they propose that revision functions should operate on belief states (also referred as epistemic states) rather than belief sets. While belief sets characterize the set of propositions that an agent holds, belief states also contains the logical consequences of those propositions, including the strategy that the agent wishes to employ at that given time. This strategy is equivalent to conditional beliefs: beliefs that one is prepared to accept conditioned on the existence of some evidence. Darwiche and Pearl formally define an epistemic state as:

Definition 2 An epistemic state Ψ is an object to which we associate a consistent propositional formula $B(\Psi)$ that denotes the current beliefs of the agent in the epistemic state Ψ . Let us denote by \mathcal{E} the set of epistemic states.

One can conclude that belief states are richer than belief sets and even that two identical belief sets can behave differently under revision, depending on how the strategy is to be modified by each new evidence.

Along with this modification to the AGM framework, that enables belief revision to be a function of epistemic states, Darwiche and Pearl also proposed four additional postulates that allow that only the conditional beliefs that don't compromise propositional beliefs will be preserved.

Darwiche and Pearl start by presenting a modification to the KM postulates earlier referred: (R1)-(R6). This modification is achieved by a weakening of postulate (R4) allowing belief revision to be a function of an epistemic state:

- (R*1) $B(\Psi \circ \mu)$ implies μ
- (R*2) If $B(\Psi) \wedge \mu$ is satisfiable, then $B(\Psi \circ \mu) \equiv B(\Psi) \wedge \mu$
- (R*3) If μ is satisfiable, then $B(\Psi \circ \mu)$ is also satisfiable
- (R*4) If $\Psi_1 = \Psi_2$ and $\mu_1 \equiv \mu_2$, then $\Psi_1 \circ \mu_1 \equiv \Psi_2 \circ \mu_2$
- (R*5) $(B(\Psi \circ \mu)) \wedge \phi$ implies $B(\Psi \circ (\mu \wedge \phi))$
- (R*6) If $(B(\Psi \circ \mu)) \wedge \phi$ is satisfiable, then $B(\Psi \circ (\mu \wedge \phi))$ implies $(B(\Psi \circ \mu)) \wedge \phi$

Where $B(\Psi)$ is a propositional sentence that defines the belief set associated to each epistemic state Ψ .

As pointed, postulate (R*4) requires the epistemic states to be identical in order to let them lead to equivalent belief sets when revised by equivalent evidence.

The four additional postulates proposed by these authors were the following:

- (C1) If $\alpha \models \mu$ then $(\Psi \circ \mu) \circ \alpha \equiv \Psi \circ \alpha$
- (C2) If $\alpha \models \neg\mu$, then $(\Psi \circ \mu) \circ \alpha \equiv \Psi \circ \alpha$
- (C3) If $B(\Psi \circ \alpha) \vdash \mu$, then $B((\Psi \circ \mu) \circ \alpha) \vdash \mu$
- (C4) If $B(\Psi \circ \alpha) \not\vdash \neg\mu$, then $B((\Psi \circ \mu) \circ \alpha) \not\vdash \neg\mu$

Postulate (C1) states that the later evidence α cannot discredit the previous evidence μ because α entails μ or, in other words, evidence α alone can yield the same belief set, making evidence μ redundant. Postulate (C2) allows the later evidence α to discredit the previous evidence μ due to the fact that α logically contradicts μ and, as before, evidence α alone would yield the same belief set. Postulate (C3) on the other hand, retains evidence μ after accommodating the more recent evidence α given that α implies μ under current beliefs. Lastly, postulate (C4) stipulates that if μ is not contradicted after seeing α then it should remain uncontradicted when α is preceded by μ itself.

Postulates (C1)-(C4) have a correspondence in terms of total preorders:

Theorem 2 [DP96] Suppose that a revision operator satisfies postulates (R*1)-(R*6).

The operator satisfies postulates (C1)-(C4) iff the operator and its corresponding faithful assignment satisfy:

- (CR1) If $\omega_1 \models \mu$ and $\omega_2 \models \mu$, then $\omega_1 \leq_{\Psi} \omega_2$ iff $\omega_1 \leq_{\Psi \circ \mu} \omega_2$
 (CR2) If $\omega_1 \models \neg\mu$ and $\omega_2 \models \neg\mu$, then $\omega_1 \leq_{\Psi} \omega_2$ iff $\omega_1 \leq_{\Psi \circ \mu} \omega_2$
 (CR3) If $\omega_1 \models \mu$ and $\omega_2 \models \neg\mu$, then $\omega_1 <_{\Psi} \omega_2$ only if $\omega_1 <_{\Psi \circ \mu} \omega_2$
 (CR4) If $\omega_1 \models \mu$ and $\omega_2 \models \neg\mu$, then $\omega_1 \leq_{\Psi} \omega_2$ only if $\omega_1 \leq_{\Psi \circ \mu} \omega_2$

According to (CR1), the order among the μ -worlds remains unchanged after revision by μ and (CR2) maintains the order among the $\neg\mu$ -worlds after revision by μ . (CR3) says that if a μ -world is strictly preferred to a $\neg\mu$ -world, then that strict preference is maintained after revision by μ . By (CR4) if a μ -world is weakly preferred to a $\neg\mu$ -world, then that weak preference is maintained after revision by μ .

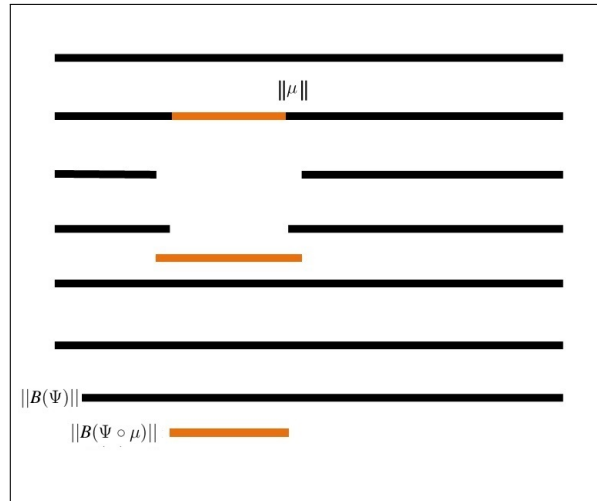


Figure 3.2: An example of a belief revision function satisfying (CR1)-(CR4).

3.2 Iterable Operators

3.2.1 Independence Postulate

Although (C1)-(C4) set the stage for iterated revision, some authors (Booth and Meyer [BM06] and Jin and Thielscher [JT07]) have indicated that these postulates are too permissive since they do not rule out operators by which all newly acquired information is given up as soon as an agent learns a fact that contradicts some of its current beliefs. In order to circumvent this they proposed the independence postulate:

- (Ind) If $\varphi \circ \alpha \not\models \neg\mu$, then $(\varphi \circ \mu) \circ \alpha \vdash \mu$

This postulate is clearly stronger than (C3) and (C4). Postulates (C1), (C2) and (Ind) were considered characteristics of a family of operators called admissible revision

sense that it only makes the minimal changes of the preorder that are needed to accept the input. In revision by μ , the minimal μ -worlds are moved to the bottom of the preorder which is otherwise left unchanged. The main characteristic of this operator is:

(Nat) If $\Psi \circ \mu \vdash \neg\alpha$, then $(\Psi \circ \mu) \circ \alpha = \Psi \circ \alpha$

And in terms of total pre-orders:

(CRNat) If $\omega_1 \notin [[\Psi \circ \mu]]$ and $\omega_2 \notin [[\Psi \circ \mu]]$, then $\omega_1 \leq_{\Psi} \omega_2 \Leftrightarrow \omega_1 \leq_{\Psi \circ \mu} \omega_2$

Lexicographic revision, also called Moderate revision, was proposed by Nayak in [Nay94b] and deeply studied by Nayak, Pagnucco and Peppas [NPP03]. When revising by μ the preorder is rearranged by putting the μ -worlds at bottom (but conserving their relative order) and the $\neg\mu$ -worlds at top (but conserving their relative order). It has the following property.

(Lex) If $\alpha \not\vdash \neg\mu$, then $(\Psi \circ \mu) \circ \alpha \vdash \mu$

Becoming the following, when using total pre-orders:

(CRLex) If $\omega_1 \models \mu$ and $\omega_2 \models \neg\mu$, then $\omega_1 <_{\Psi \circ \mu} \omega_2$

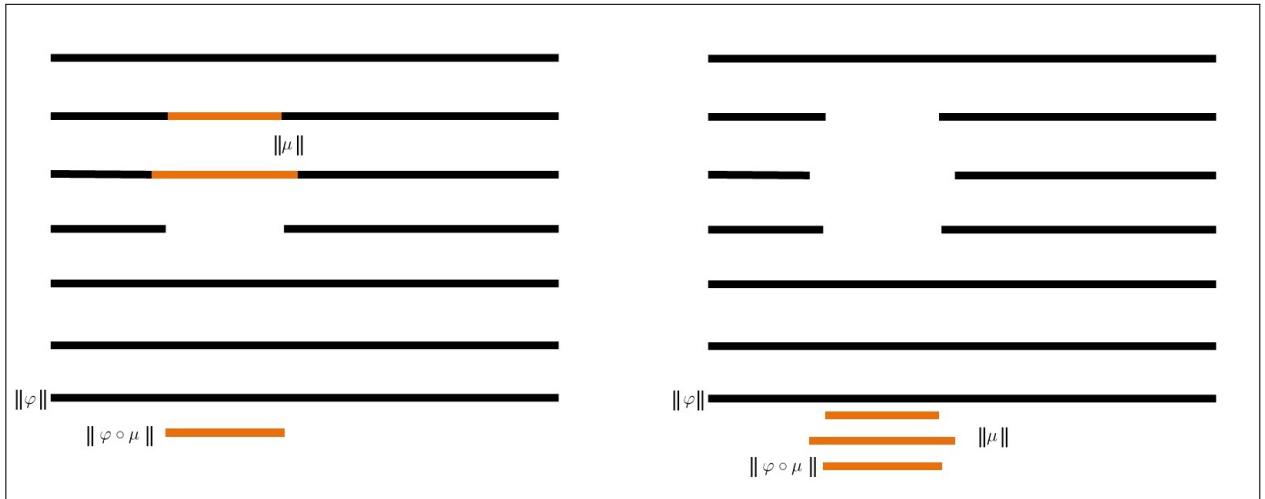


Figure 3.4: An example of Natural Revision (left) and Lexicographic revision (right).

Rott [Rot09] identified a third type of iterable revision operators, known as Radical revision, that we are not addressing in this dissertation.

Chapter 4

The Katsuno and Mendelzon Update

4.1 Definition

In their 1992 paper [KM92] Hirofumi Katsuno and Albert O. Mendelzon demonstrate an important result. They make a formal and fundamental distinction between two kinds of modifications that can be applied to a knowledge base: revision and update.

While revision is used when obtaining new information about a static world, that is, changes take place at the knowledge level, update brings the knowledge base up to date when the world described by it changes.

Intuitively one can understand the difference between the postulates for revision and update as follows:

Suppose that we have a knowledge base φ and we want to revise it with the sentence μ . Using revision methods, which satisfy the AGM postulates, the system is going to select from all the models of μ those that are closest to the models of φ . The theory $\varphi \circ \mu$ is then defined by these selected models.

When speaking of update methods, what we have is a selection that chooses, for every model M of the knowledge base φ , the set of models that are closest to M : the newly obtained theory $\varphi \diamond \mu$ (where \diamond represents the update operator) describes the union of all such models.

Grahne [Gra91] explained update the following way:

“Since we are confined to our set of possibilities, we must make the change come true in all of our candidate worlds. Semantically, we change each of the possible worlds ‘as little as possible’ in order to make the new state of affairs hold. Our new syntactic description of the worlds of interest should now correctly reflect the outcome of this set of changes. The function that maps the old description to the new is called an update.”

Keeping in mind all that was mentioned earlier about revision and its postulates, Katsuno and Mendelzon presented the following postulates for update [KM92]:

- (U1) $\varphi \diamond \mu$ implies μ
- (U2) If φ implies μ then $\varphi \diamond \mu$ is equivalent to φ
- (U3) If both φ and μ are satisfiable then $\varphi \diamond \mu$ is also satisfiable
- (U4) If $\varphi_1 \equiv \varphi_2$ and $\mu_1 \equiv \mu_2$ then $\varphi_1 \diamond \mu_1 \equiv \varphi_2 \diamond \mu_2$
- (U5) $(\varphi \diamond \mu) \wedge \phi$ implies $\varphi \diamond (\mu \wedge \phi)$
- (U6) If $\varphi \diamond \mu_1$ implies μ_2 and $\varphi \diamond \mu_2$ implies μ_1 then $\varphi \diamond \mu_1 \equiv \varphi \diamond \mu_2$
- (U7) If φ is complete then $(\varphi \diamond \mu_1) \wedge (\varphi \diamond \mu_2)$ implies $\varphi \diamond (\mu_1 \vee \mu_2)$
- (U8) $(\varphi_1 \vee \varphi_2) \diamond \mu \equiv (\varphi_1 \diamond \mu) \vee (\varphi_2 \diamond \mu)$

We can observe that postulates (U1)-(U5) directly correspond to the postulates (R1)-(R5) presented in chapter 2.

The following theorem, also proved by Katsuno and Mendelzon, shows that all update operators defined by a partial pre-order can be captured by the proposed postulates:

Theorem 3 [KM92] *Let \diamond be an update operator. The following conditions are equivalent:*

1. The update operator \diamond satisfies conditions (U1)-(U8);
2. There exists a faithful assignment that maps each interpretation I to a partial pre-order \leq_I such that

$$Mod(\varphi \diamond \mu) = \bigcup_{I \in Mod(\varphi)} Min(Mod(\mu), \leq_I)$$

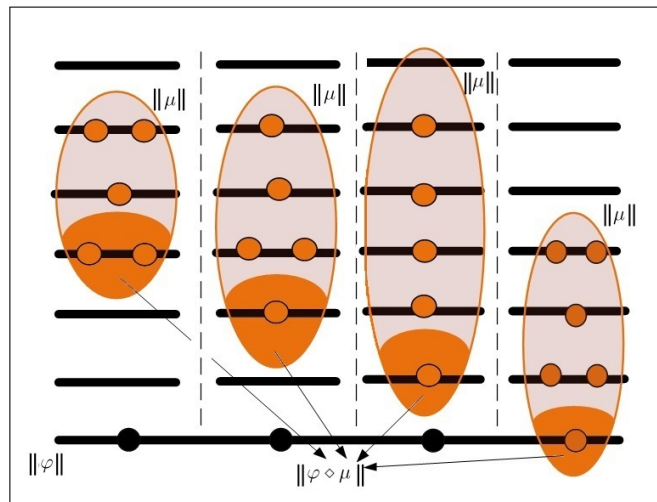


Figure 4.1: An example of an update operator

4.2 Update Operators

Having presented the definition of Update according to Katsuno and Mendelzon, and similarly to what was presented in Section 2.4, in this section we present some proposals for update operators that can be found in the literature.

These operators have different strengths and complexity and a more thorough description can be found in the works of Andreas Herzig and Omar Rifi [HR98, HR99].

4.2.1 PMA

Winslett [Win88] introduced the Possible Models Approach (PMA) in 1988, in the context of reasoning about action and change. This operator is based on the minimization of the distance between interpretations. It is interesting to note that Winslett's work is prior to Katsuno and Mendelzon's concept of update.

Let φ be the belief base and μ the new belief, then the PMA update operator is described as:

$$Mod(\varphi \diamond_{pma} \mu) = \bigcup_{I \in Mod(\varphi)} Min(Mod(\mu), I)$$

Where $Mod(\mu)$ represents the set of models of the formula μ and $Min(Mod(\mu), I)$ the subset of models of μ that are closest to I .

In this model the proximity relation between two models I_1 and I_2 and an interpretation I is defined by a partial order \leq_I such that $I_1 \leq_I I_2$ if and only if $diff(I, I_1) \subseteq diff(I, I_2)$. $diff(A, B)$ represents the set of propositions in which A and B differ.

This said, $J \in Min(Mod(\mu), I)$ if and only if J is minimal in $Mod(\mu)$ regarding \leq_I , that is, it doesn't exist $J' \in Mod(\mu)$ such that $J' \leq_I J$.

It can be proved that the PMA update operator satisfies postulates (U1)-(U8).

4.2.2 FORBUS

The operator proposed by Forbus [For89] is stronger than the PMA operator and is the update counterpart of Dalal's semantics for belief revision (see section 2.4.1).

Forbus operator calculates distances and preferences between the models of the belief base φ and the new information μ . Let w be a model of φ and u_1, u_2 two models of μ . The total order \leq_w between models is given by:

$$u_i \leq_w u_j \text{ iff } |dist(w, u_i)| \leq |dist(w, u_j)|,$$

where $|dist(w, u)|$ represents the cardinality of the distance between the models. The models are selected according to this relation and its union constitutes the new belief base.

This update operator also satisfies postulates (U1)-(U8).

4.2.3 MCD

In 1996 Zhang and Foo [ZF96] proposed the MCD (Minimal Change with Disjunctive information) operator which deals with the problem of disjunction in the PMA model.

Let φ be a knowledge base and μ a propositional formula. The update operator defined by Zhang and Foo, $\varphi \diamond_{MCD} \mu$, is defined by:

- (1) $\varphi \diamond_{MCD} \mu = \varphi$ if φ entails μ or φ is inconsistent
- (2) $Mods(\varphi \diamond_{MCD} \mu) = \bigcup_{S \in Models(\varphi)} Res(S, \mu)^{MCD}$, otherwise

Where $Res(S, \mu)^{MCD}$ represents the set of all possible states of the world resulting from updating a state of the world, S , with μ , using the MCD model.

The MCD update operator doesn't satisfy postulates (U5) and (U7), although it satisfies all the remaining ones.

Chapter 5

Iteration for KM-update

5.1 Definition

As we mentioned, a KM-update operator \diamond is defined for all the possible belief sets φ . Consequently, in a first view, iteration does not seem to require a special attention, since $(\varphi \diamond \alpha) \diamond \beta$ is well defined. However, what happens if we want to make changes in our preferences as a consequence of updating by α ? In that case we need to define a new kind of operator \diamond_α to reflect these changes. This implies that, $(\varphi \diamond \alpha) \diamond \beta$ and $\varphi \diamond_\alpha \beta$ will be, in general, different operations.

In order to differentiate these new operators from the original KM operators we will use the notation \diamond_s , where $s = \{\alpha_1, \dots, \alpha_n\}$ is a sequence of updates. If $s = \emptyset$, we will denote \diamond , if $s = \{\alpha\}$ we will denote \diamond_α . (U1)-(U8) will remain unchanged for the new class of operators.

In order to define iteration of update we start by adapting the Darwiche and Pearl's (C1)-(C4) postulates:

- (CU1) If $\alpha \vdash \mu$ then $\varphi \diamond_\mu \alpha \equiv \varphi \diamond \alpha$
- (CU2) If $\alpha \vdash \neg\mu$ then $\varphi \diamond_\mu \alpha \equiv \varphi \diamond \alpha$
- (CU3) If $\varphi \diamond \alpha \vdash \mu$ then $\varphi \diamond_\mu \alpha \vdash \mu$
- (CU4) If $\varphi \diamond \alpha \not\vdash \neg\mu$ then $\varphi \diamond_\mu \alpha \not\vdash \neg\mu$

Basically, the only change needed is to replace the revision operator by update and readjust the notation. In this case, all the additional structure that was needed in belief revision, resides in the operator \diamond . Consequently, a belief state can be defined by the pair (φ, \diamond) .

In terms of faithful assignment, the corresponding (CR) properties, where the definition of faithful assignment must be adapted to belief states (i.e., by using $\leq_{\{\bullet, \omega\}}$ instead of \leq_ω), are:

- (CRU1) If $\omega_1 \models \mu$ and $\omega_2 \models \mu$, then $\omega_1 \leq_{\{\bullet, \omega\}} \omega_2 \Leftrightarrow \omega_1 \leq_{\{\bullet, \mu, \omega\}} \omega_2$
- (CRU2) If $\omega_1 \models \neg\mu$ and $\omega_2 \models \neg\mu$, then $\omega_1 \leq_{\{\bullet, \omega\}} \omega_2 \Leftrightarrow \omega_1 \leq_{\{\bullet, \mu, \omega\}} \omega_2$

(CRU3) If $\omega_1 \models \mu$ and $\omega_2 \models \neg\mu$, then $\omega_1 <_{\{\bullet, \omega\}} \omega_2$ implies $\omega_1 <_{\{\bullet, \mu, \omega\}} \omega_2$

(CRU4) If $\omega_1 \models \mu$ and $\omega_2 \models \neg\mu$, then $\omega_1 \leq_{\{\bullet, \omega\}} \omega_2$ implies $\omega_1 \leq_{\{\bullet, \mu, \omega\}} \omega_2$

(CRU1)-(CRU2) require that the order among μ -worlds and the order among the $\neg\mu$ -worlds remains unchanged after update by μ in all of the preorders defined for each ω_i . In the same way, (CRU3) says that if a μ -world is strictly preferred to a $\neg\mu$ -world, then that strict preference is maintained after revision by μ in all of the preorders defined for each ω_i and finally (CRU4) says that if a μ -world is weakly preferred to a $\neg\mu$ -world, then that weak preference is maintained after update by μ in all of the preorders defined for each ω_i .

The next theorem proves our last assertion.

Proposition 1 \diamond_s is an update operator if and only if there exists a faithful assignment that maps each possible world ω to a partial pre-order $<_{\{\bullet, s, \omega\}}$ such that:

$$[[\varphi \diamond_s \alpha]] = \bigcup_{\omega \models \varphi} \min([[\alpha]], \leq_{\{\bullet, s, \omega\}})$$

Theorem 4 Let \diamond_s be an update operator. Then:

1. \diamond_s satisfies (CU1) iff its corresponding faithful assignment satisfies (CRU1)
2. \diamond_s satisfies (CU2) iff its corresponding faithful assignment satisfies (CRU2)
3. \diamond_s satisfies (CU3) iff its corresponding faithful assignment satisfies (CRU3)
4. \diamond_s satisfies (CU4) iff its corresponding faithful assignment satisfies (CRU4)

Proof:¹

(CU1) \Leftrightarrow (CRU1)

(\Rightarrow) Assume (CU1) holds and let $\omega_1 \models \mu$ and $\omega_2 \models \mu$. Let $\alpha \equiv \alpha_{\{\omega_1, \omega_2\}}$. Then $\alpha \vdash \mu$ and due to (CU1) $\varphi \diamond_\mu \alpha \equiv \varphi \diamond \alpha$. Hence $\min(\{\omega_1, \omega_2\}, \leq_{\{\bullet, \omega\}}) = \min(\{\omega_1, \omega_2\}, \leq_{\{\bullet, \mu, \omega\}})$ from which it follows that $\omega_1 \leq_{\{\bullet, \omega\}} \omega_2 \Leftrightarrow \omega_1 \leq_{\{\bullet, \mu, \omega\}} \omega_2$.

(\Leftarrow) Assume (CRU1) holds and let $\alpha \vdash \mu$. We want to show that $\varphi \diamond_\mu \alpha \equiv \varphi \diamond \alpha$. Condition (CRU1) implies that $\leq_{\{\bullet, \omega\}}$ and $\leq_{\{\bullet, \mu, \omega\}}$ are equivalent for all $\omega' \in [[\alpha]]$ since $[[\alpha]] \subseteq [[\mu]]$. Hence:

$$\begin{aligned} [[\varphi \diamond \alpha]] &= \bigcup_{\omega \models \varphi} \min([[\alpha]], \leq_{\{\bullet, \omega\}}) \\ [[\varphi \diamond_\mu \alpha]] &= \bigcup_{\omega \models \varphi} \min([[\alpha]], \leq_{\{\bullet, \mu, \omega\}}) \\ [[\varphi \diamond \alpha]] &= [[\varphi \diamond_\mu \alpha]] \end{aligned}$$

(CU2) \Leftrightarrow (CRU2). The proof is symmetric with the one above.

¹Adapted from [[DP96], Proof of Theorem 13]. For the sake of simplicity, we will use in the proofs \diamond instead of \diamond_s , since the subscript is not necessary for the proofs.

(CU3) \Leftrightarrow (CRU3)

(\Rightarrow) Assume (CU3) holds and let $\omega_1 \models \mu$, $\omega_2 \models \neg\mu$ and $\omega_1 <_{\{\bullet, \omega\}} \omega_2$. Let $\alpha \equiv \alpha_{\{\omega_1, \omega_2\}}$. Then $[[\alpha_\omega \diamond \alpha]] = \min([[\alpha]], \leq_{\{\bullet, \omega\}}) = \{\omega_1\}$, from which it follows that $\alpha_\omega \diamond \alpha \vdash \mu$. By (CU3) $\alpha_\omega \diamond_\mu \alpha \vdash \mu$, from which it follows that $[[\alpha_\omega \diamond_\mu \alpha]] = \min([[\alpha]], \leq_{\{\bullet, \mu, \omega\}}) \subseteq [[\mu]]$, hence $[[\alpha_\omega \diamond_\mu \alpha]] = \{\omega_1\}$, from which we can conclude that $\omega_1 <_{\{\bullet, \mu, \omega\}} \omega_2$.

(\Leftarrow) Assume (CRU3) holds and let $\varphi \diamond \alpha \vdash \mu$. From $[[\varphi \diamond \alpha]] = \bigcup_{\omega \models \varphi} \min([[\alpha]], \leq_{\{\bullet, \omega\}})$ it follows that for all $\omega \models \varphi$ if $\omega' \in \min([[\alpha]], \leq_{\{\bullet, \omega\}})$ implies that $\omega' \models \alpha \wedge \mu$ and for all $\omega'' \models \alpha \wedge \neg\mu$ it follows that $\omega' <_{\{\bullet, \omega\}} \omega''$. (CRU3) yields $\omega' <_{\{\bullet, \mu, \omega\}} \omega''$ for all $\omega'' \models \alpha \wedge \neg\mu$, hence $\omega'' \notin \min([[\alpha]], \leq_{\{\bullet, \mu, \omega\}})$. Since this is valid for all $\omega \models \varphi$ we can conclude that $\varphi \diamond_\mu \alpha \vdash \mu$.

(CU4) \Leftrightarrow (CRU4)

(\Rightarrow) Assume (CU4) holds and let $\omega_1 \models \mu$, $\omega_2 \models \neg\mu$ and $\omega_1 \leq_{\{\bullet, \omega\}} \omega_2$. Let $\alpha \equiv \alpha_{\{\omega_1, \omega_2\}}$. Then $\omega_1 \in [[\alpha_\omega \diamond \alpha]] = \min([[\alpha]], \leq_{\{\bullet, \omega\}})$, from which it follows that $\alpha_\omega \diamond \alpha \not\vdash \neg\mu$. By (CU4) $\alpha_\omega \diamond_\mu \alpha \not\vdash \neg\mu$, from which it follows that $[[\alpha_\omega \diamond_\mu \alpha]] \cap [[\mu]] \neq \emptyset$, i.e., $\min([[\alpha]], \leq_{\{\bullet, \mu, \omega\}}) \cap [[\mu]] \neq \emptyset$, hence $\omega_1 \in [[\alpha_\omega \diamond_\mu \alpha]]$, from which we can conclude that $\omega_1 \leq_{\{\bullet, \mu, \omega\}} \omega_2$.

(\Leftarrow) Assume (CRU4) holds and let $\varphi \diamond \alpha \not\vdash \neg\mu$. From $[[\varphi \diamond \alpha]] = \bigcup_{\omega \models \varphi} \min([[\alpha]], \leq_{\{\bullet, \omega\}})$ it follows that there exists v such that $v \models \varphi$ and for some $\omega' \in \min([[\alpha]], \leq_{\{\bullet, v\}})$ it holds that $\omega' \models \alpha \wedge \mu$ and for all $\omega'' \models \alpha \wedge \neg\mu$ it follows that $\omega' \leq_{\{\bullet, v\}} \omega''$. (CRU4) yields $\omega' \leq_{\{\bullet, \mu, v\}} \omega''$ for all $\omega'' \models \alpha \wedge \neg\mu$, hence $\omega' \in \min([[\alpha]], \leq_{\{\bullet, \mu, v\}})$, from which it follows that $\omega' \in \bigcup_{\omega \models \varphi} \min([[\alpha]], \leq_{\{\bullet, \mu, \omega\}})$. Hence $\varphi \diamond_\mu \alpha \not\vdash \neg\mu$. ■

At this point, an important difference between iteration of revision and iteration of update appears. In belief revision, AGM, without the iteration postulates, didn't provide any insight about the new preorder. On the other hand, \diamond_α , without any change w.r.t. $\diamond_{\{\}}$ is well defined and satisfies (CU1)-(CU4):

Proposition 2 *Let \diamond_s be an update operator and let $\diamond_\alpha = \diamond_{\{\}}$. Then \diamond_α satisfies (CU1)-(CU4).*

For that reason, the claims pointed out by Booth and Meyer [BM06] and Jin and Thielscher [JT07] gain a new importance in update. The corresponding *independence postulate* for iterated update is

(U-Ind) If φ is a complete formula and $\varphi \diamond \alpha \not\vdash \neg\mu$, then $\varphi \diamond_\mu \alpha \vdash \mu$

which corresponds, in terms of faithful assignments, to the following property (see an example in Figure 5.1):

(CRUInd) If $\omega_1 \models \mu$ and $\omega_2 \models \neg\mu$, then $\omega_1 \leq_{\{\bullet, \omega\}} \omega_2 \Rightarrow \omega_1 <_{\{\bullet, \mu, \omega\}} \omega_2$

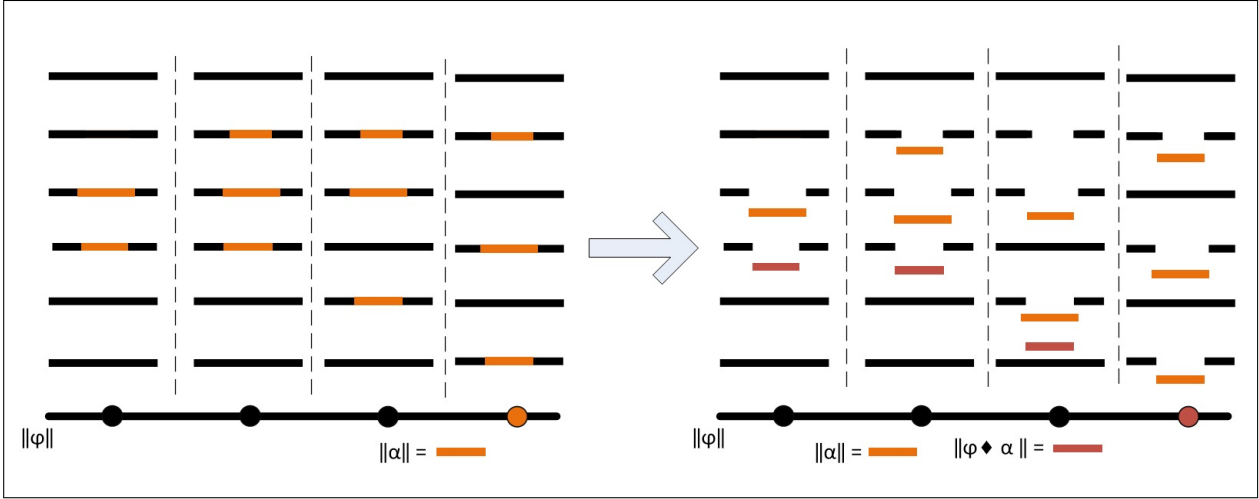


Figure 5.1: An example of an update operator satisfying (CU1)-(CU4) and (C-Ind) of $[[\varphi \diamond \alpha]]$.

Theorem 5 Let \diamond_s be an update operator. Then \diamond_s satisfies (U-Ind) iff its corresponding faithful assignment satisfies (CRUInd).

Proof:

(U-Ind) \Leftrightarrow (CRUInd)

(\Rightarrow) Assume (U-Ind) holds and let $\omega_1 \models \mu$, $\omega_2 \models \neg\mu$ and $\omega_1 \leq_{\{\bullet, \omega\}} \omega_2$. Let $\alpha \equiv \alpha_{\{\omega_1, \omega_2\}}$. Then $\omega_1 \in [[\alpha_\omega \diamond \alpha]] = \min([[\alpha]], \leq_{\{\bullet, \omega\}})$, from which it follows that $\alpha_\omega \diamond \alpha \not\models \neg\mu$. By (U-Ind) $\alpha_\omega \diamond_\mu \alpha \vdash \mu$, from which it follows that $[[\alpha_\omega \diamond_\mu \alpha]] = \min([[\alpha]], \leq_{\{\bullet, \omega\}}) \subseteq [[\mu]]$, hence $[[\alpha_\omega \diamond_\mu \alpha]] = \{\omega_1\}$, from which we can conclude that $\omega_1 <_{\{\bullet, \omega\}} \omega_2$.

(\Leftarrow) Assume (CRUInd) holds and let φ be a complete formula such that $\varphi \diamond \alpha \not\models \neg\mu$. Since φ is complete, there exists some v such that $\varphi \equiv \alpha_v$. Then $[[\varphi \diamond \alpha]] = \min([[\alpha]], \leq_{\{\bullet, v\}})$. Due to $\varphi \diamond \alpha \not\models \neg\mu$ it follows that there exists $\omega' \in \min([[\alpha]], \leq_{\{\bullet, \omega\}})$ such that $\omega' \models \alpha \wedge \mu$ and for all $\omega'' \models \alpha \wedge \neg\mu$ it follows that $\omega' \leq_{\{\bullet, v\}} \omega''$. (CRUInd) yields $\omega' <_{\{\bullet, v\}} \omega''$ for all $\omega'' \models \alpha \wedge \neg\mu$, hence $\omega' \in \min([[\alpha]], \leq_{\{\bullet, v\}})$ and there ω'' not exists such that $\omega'' \models \alpha \wedge \neg\mu$ and $\omega'' \in \min([[\alpha]], \leq_{\{\bullet, v\}})$. Hence $\varphi \diamond_\mu \alpha \vdash \mu$. ■

Proposition 3 Let \diamond be an update operator satisfying (C-Ind). Then, in general, $\diamond_\alpha \neq \diamond_{\{\}}$.

5.2 Two families of KM-update

In this subsection we show how to define (as in belief revision) natural and lexicographic update. First we need to adapt the postulates formerly defined:

(U-Nat) If φ is a complete formula and $\varphi \diamond \mu \vdash \neg\alpha$, then $\varphi \diamond_{\mu} \alpha \equiv \varphi \diamond \alpha$

(U-Lex) If $\varphi \vdash \neg\alpha$ and $\alpha \not\vdash \neg\mu$, then $\varphi \diamond_{\mu} \alpha \vdash \mu$

and then provide their correspondent postulates in terms of possible worlds:

(CRUNat) If $\omega_1, \omega_2 \models \neg(\varphi \diamond \mu)$, then $\omega_1 \leq_{\{\bullet, \omega\}} \omega_2 \Leftrightarrow \omega_1 \leq_{\{\bullet, \mu, \omega\}} \omega_2$

(CRULex) If $\omega_1 \models \mu$ and $\omega_2 \models \neg\mu$, then $\omega_1 <_{\{\bullet, \mu, \omega\}} \omega_2$ for all $\omega \neq \omega_2$

The following representation theorem shows the equivalences:

Theorem 6 *Let \diamond be an update operator. Let f be its corresponding faithful assignment, i.e., such that*

$$[[\varphi \diamond \alpha]] = \bigcup_{\omega \models \varphi} \min([[\alpha]], \leq_{\{\bullet, \omega\}})$$

Then:

1. \diamond satisfies (U-Nat) iff the faithful assignment satisfies (CRUNat)
2. \diamond satisfies (U-Lex) iff the faithful assignment satisfies (CRULex)

Proof:

(U-Nat) \Leftrightarrow (CRUNat)

(\Rightarrow) Assume (U-Nat) holds and let $\omega_1, \omega_2 \models \neg(\varphi \diamond \mu)$. Let $\alpha \equiv \alpha_{\{\omega_1, \omega_2\}}$. Then $\alpha \vdash \neg\varphi \diamond \mu$ from which it follows that $\varphi \diamond \mu \vdash \neg\alpha$ and (by U-Nat) $\varphi \diamond_{\mu} \alpha \equiv \varphi \diamond \alpha$. Since φ is a complete formula, there exists v such that $\alpha_v \equiv \varphi$. Hence $\min(\{\omega_1, \omega_2\}, \leq_{\{\bullet, v\}}) = \min(\{\omega_1, \omega_2\}, \leq_{\{\bullet, \mu, v\}})$ from which it follows that $\omega_1 \leq_{\{\bullet, v\}} \omega_2 \Leftrightarrow \omega_1 \leq_{\{\bullet, \mu, v\}} \omega_2$.

(\Leftarrow) Assume (CRUNat) holds and let φ be a complete formula such that $\varphi \diamond \mu \vdash \neg\alpha$. Since φ is complete there exists v such that $\varphi \equiv \alpha_v$. Then $[[\varphi \diamond \alpha]] = \min([[\alpha]], \leq_{\{\bullet, v\}})$. We have that $[[\alpha]] \models \neg(\varphi \diamond \mu)$. Condition (CRUNat) implies that $\leq_{\{\bullet, v\}}$ and $\leq_{\{\bullet, \mu, v\}}$ are equivalent for all $\omega' \in [[\alpha]]$. Hence:

$$\begin{aligned} [[\varphi \diamond \alpha]] &= \min([[\alpha]], \leq_{\{\bullet, v\}}) \\ [[\varphi \diamond \alpha]] &= \min([[\alpha]], \leq_{\{\bullet, \mu, v\}}) \\ [[\varphi \diamond \alpha]] &= [[\varphi \diamond_{\mu} \alpha]] \end{aligned}$$

(U-Lex) \Leftrightarrow (CRULex)

(\Rightarrow) Assume (U-Lex) holds and let $\omega_1 \models \mu$ and $\omega_2 \models \neg\mu$. Let $\alpha \equiv \alpha_{\{\omega_1, \omega_2\}}$. Then $\alpha \not\vdash \neg\mu$. Let φ be a complete formula such that $\varphi \vdash \neg\alpha$. Then $\varphi \not\equiv \alpha_{\omega_2}$. Then it

follows by (U-Lex) that $\varphi \diamond_{\mu} \alpha \vdash \mu$. Since φ is complete there exists $v \neq \omega_2$ such that $\varphi \equiv \alpha_v$. Then $\llbracket \varphi \diamond_{\mu} \alpha \rrbracket = \min(\llbracket \alpha \rrbracket, \leq_{\{\diamond_{\mu}, v\}})$. Since $\llbracket \varphi \diamond_{\mu} \alpha \rrbracket \subseteq \llbracket \mu \rrbracket$ it follows that $\min(\llbracket \alpha \rrbracket, \leq_{\{\diamond_{\mu}, v\}}) = \{\omega_1\}$. Hence $\omega_1 <_{\{\diamond_{\mu}, v\}} \omega_2$.

(\Leftarrow) Assume (CRULex) holds and let φ , α and μ such that $\varphi \vdash \neg\alpha$ and $\alpha \not\vdash \neg\mu$. $\varphi \equiv \alpha_{\{\omega_1, \dots, \omega_n\}}$, for $\omega_1, \dots, \omega_n \in W$. Then $\{\omega_1, \dots, \omega_n\} \subseteq \llbracket \neg\alpha \rrbracket$. Then $\omega_i \notin \min(\llbracket \alpha \rrbracket, \leq_{\{\diamond_{\mu}, \omega_i\}})$, for $i = 1, \dots, n$. It follows from (CRULex) that $\omega_j <_{\{\diamond_{\mu}, \omega_i\}} \omega_k$ for all $\omega_j \models \mu$ and $\omega_k \models \neg\mu$, $k \neq i$ and $i = 1, \dots, n$. Then $\min(\llbracket \alpha \rrbracket, \leq_{\{\diamond_{\mu}, \omega_i\}}) \cap \llbracket \neg\mu \rrbracket = \emptyset$. Hence $\varphi \diamond_{\mu} \alpha \vdash \mu$. ■

Chapter 6

Conclusion

The main purpose of this dissertation was to propose a model for iteration of update, using the definition of this type of modification given by Katsuno and Mendelzon in their work.

Perhaps a question that may arise is whether iterated update is worth investigating. We believe it is! Iterated update is of the most importance when used in A.I. and there are not so many works on this theme as there are on iterated revision.

This dissertation started with the definition of the AGM theory: the basic work than broadened the study of belief change. And throughout out it we tried to summarize all the key subjects that allowed us to construct our model.

Originated in two large research areas, philosophy and computer science, the belief change study has become the subject of the work of a considerable number of investigators. Several extensions and generalizations have been suggested as well as important applications and connections with other areas.

One of this extensions is iterated change: the AGM framework does not ensure the rational preservation of conditional beliefs. In order to allow the sequential revision of beliefs in response to a string of observations, Darwiche and Pearl proposed four new postulates that have proven to be accurate, when used together with a subtle but major transformation by which the revision functions operate on belief states rather than belief sets.

Our goal was to adapt the Darwiche and Pearl's postulates for iterated belief revision to the case of iterated update.

According to the proposed definition, it is possible to create a sequence of updates that play an important role on how the new belief state will be updated.

There is, however, a big difference in simply applying the original update operator to the updated belief set or using the approach we propose: formally, $(\varphi \diamond \alpha) \diamond \beta$ and $\varphi \diamond_{\alpha} \beta$ can be different.

Example 1 Let $\mathcal{P} = \{\alpha, \beta\}$ and the correspondent possible worlds $W = \{\omega_1, \omega_2, \omega_3, \omega_4\}$ defined as

$$\begin{aligned}\omega_1 &= \llbracket \{\alpha, \beta\} \rrbracket. \\ \omega_2 &= \llbracket \{\alpha, \neg\beta\} \rrbracket. \\ \omega_3 &= \llbracket \{\neg\alpha, \beta\} \rrbracket. \\ \omega_4 &= \llbracket \{\neg\alpha, \neg\beta\} \rrbracket.\end{aligned}$$

Let \diamond be an update operator satisfying (CU1)-(CU4) and (U-Ind). Let $\varphi \equiv \neg\beta$, i.e., $\llbracket \varphi \rrbracket = \{\omega_2, \omega_4\}$. Consider the total preorders represented in Figure 5.1. Then:

$$\begin{aligned}\llbracket \varphi \diamond \alpha \rrbracket &= \{\omega_2\}. \\ \llbracket (\varphi \diamond \alpha) \diamond \beta \rrbracket &= \{\omega_1, \omega_3\}. \\ \llbracket \varphi \diamond_\alpha \beta \rrbracket &= \{\omega_1\}.\end{aligned}$$

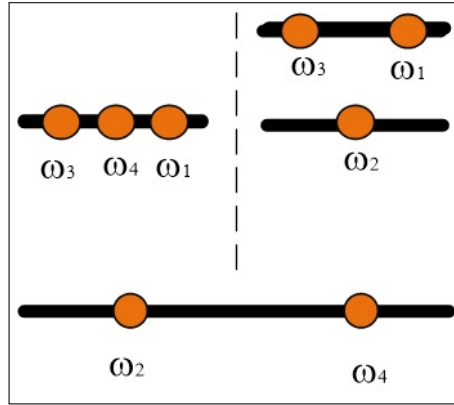


Figure 6.1: The total preorders associated to Example 1.

By using a new operator \diamond_α we are able to reflect the changes in our preferences as a consequence of updating by α and with the proposed postulates a consistent outcome is presumedly achieved.

Postulates (CRU1)-(CRU4), (CRUInd), (CRUNat) and (CRULex), which allow for the definition of large families of update operators, were also defined. New postulates can, obviously, be added in order to specify a new operator. For instance, we can define an update operator \diamond that satisfies (CRU1)-(CRU2), (CRUInd) and

(CRUSoft) If $\omega_1 \models \mu, \omega_2 \models \neg\mu$ then $\omega_2 <_{\{\cdot, \omega\}} \omega_1 \Rightarrow \omega_2 \leq_{\{\cdot, \mu, \omega\}} \omega_1$

This postulate only allows for little (soft) changes in the preorder (this property comes from improvement operators [KP08]). If a model of $\neg\mu$ is just a little more plausible than a model of μ regarding ω then, after an update, the two models will have the same plausibility.

The use of different postulates can help in the definition of different behaviors. For instance, the previous update function satisfies $\diamond_{\{\alpha, \beta\}} = \diamond_{\{\beta, \alpha\}}$.

In this dissertation only functions and postulates adapted from belief revision to update were studied. However, due to the intrinsic differences between update and revision, we believe an interesting future point is to explore new proper iterated update functions.

The results of this work will be published in the book “Foundations of Formal Rationality: Essays Dedicated to Gabriele Kern-Isberner on the Occasion of Her 60th Birthday” of the College Publications Tribute series.

Bibliography

- [AB01] Carlos Areces and Verónica Becher. Iterable AGM functions. In H.Rott and M-A Williams, editors, *Frontiers in Belief Revision*, Applied Logic Series, pages 261–277. Kluwer Academic Publishers, 2001.
- [AGM85] Carlos Alchourrón, Peter Gärdenfors, and David Makinson. On the logic of theory change: Partial meet contraction and revision functions. *Journal of Symbolic Logic*, 50:510–530, 1985.
- [AM81] Carlos Alchourrón and David Makinson. Hierarchies of regulations and their logic. In Risto Hilpinen, editor, *New Studies in Deontic Logic: Norms, Actions, and the Foundations of Ethics*, pages 125–148, 1981.
- [AM82] Carlos Alchourrón and David Makinson. On the logic of theory change: Contraction functions and their associated revision functions. *Theoria*, 48:14–37, 1982.
- [BM06] Richard Booth and Thomas Meyer. Admissible and restrained revision. *Journal of Artificial Intelligence Research*, 26:127–151, 2006.
- [Bor85] Alexander Borgida. Language features for flexible handling of exceptions in information systems. *ACM Trans. Database Syst.*, 10(4):565–603, 1985.
- [Bou93] Craig Boutilier. Revision sequences and nested conditionals. In *Proc. 13th Int. Joint Conf. on Artificial Intelligence (IJCAI'93)*, pages 519–525, 1993.
- [Bou96] Craig Boutilier. Iterated revision and minimal change of conditional beliefs. *Journal of Philosophical Logic*, 25:263–305, 1996.
- [Bre91] Gerhard Brewka. Belief revision in a framework for default reasoning. In Fuhrmann and Morreau, editors, *The Logic of Theory Change*, pages 206–222, Berlin, 1991. Springer-Verlag.
- [Dal88a] Mukesh Dalal. Investigations into a theory of knowledge base revision: Preliminary report. In *Seventh National Conference on Artificial Intelligence (AAAI-88)*, pages 475–479, St. Paul, 1988.
- [Dal88b] Mukesh Dalal. Updates in propositional databases. Technical Report DCS-TR-222, Rutgers University (New Brunswick, NJ US), 1988.
- [DP96] Adnan Darwiche and Judea Pearl. On the logic of iterated belief revision. *Artificial intelligence*, 89:1–29, 1996.

- [FH11] Eduardo Fermé and Sven Ove Hansson. AGM 25 years: Twenty-five years of research in belief change. *Journal of Philosophical Logic*, 40:295–331, 2011.
- [For89] Kenneth D. Forbus. Introducing actions into qualitative simulation. In N. S. Sridharan, editor, *IJCAI*, pages 1273–1278. Morgan Kaufmann, 1989.
- [Gär78] Peter Gärdenfors. Conditionals and changes of belief. *Acta Philosophica Fennica*, 30:381–404, 1978.
- [Gär82] Peter Gärdenfors. Rules for rational changes of belief. In Tom Pauli, editor, *Philosophical Essays dedicated to Lennart Åqvist on his fiftieth birthday*, number 34 in *Philosophical Studies*, pages 88–101, 1982.
- [Gär88] Peter Gärdenfors. *Knowledge in Flux: Modeling the Dynamics of Epistemic States*. The MIT Press, Cambridge, 1988.
- [Gra91] Gösta Grahne. Updates and counterfactuals. In *Proceedings of the 2nd International Conference on Principles of Knowledge Representation and Reasoning (KR'91)*. Cambridge, MA, USA, April 22-25, 1991., pages 269–276, 1991.
- [GS88a] Matthew L. Ginsberg and David E. Smith. Reasoning about action I: A possible worlds approach. *Artif. Intell.*, 35(2):165–195, 1988.
- [GS88b] Matthew L. Ginsberg and David E. Smith. Reasoning about action II: the qualification problem. *Artif. Intell.*, 35(3):311–342, 1988.
- [Har77] William Harper. Rational conceptual change. In The University of Chicago Press, editor, *PSA: Proceedings of the Biennial Meeting of the Philosophy of Science Association, Volume Two: Symposia and Invited Papers*, pages 462–494, 1977.
- [HR98] Andreas Herzig and Omar Rifi. Update operations: A review. In *Proceedings of the 13th European Conference on Artificial Intelligence (ECAI-98)*, pages 13–17. John Wiley & Sons, Ltd, 1998.
- [HR99] Andreas Herzig and Omar Rifi. Propositional belief base update and minimal change. *Artificial Intelligence*, 115:107–138, 1999.
- [JT07] Yi Jin and Michael Thielscher. Iterated belief revision, revised. *Artificial Intelligence*, 171:1–18, 2007.
- [KM89] Hirofumi Katsuno and Alberto O. Mendelzon. A unified view of propositional knowledge base updates. In N. S. Sridharan, editor, *IJCAI*, pages 1413–1419. Morgan Kaufmann, 1989.
- [KM91] Hirofumi Katsuno and Alberto Mendelzon. Propositional knowledge base revision and minimal change. *Journal of Artificial Intelligence*, 52:263–294, 1991.

- [KM92] Hirofumi Katsuno and Alberto Mendelzon. On the difference between updating a knowledge base and revising it. In Peter Gärdenfors, editor, *Belief Revision*, number 29 in Cambridge Tracts in Theoretical Computer Science, pages 183–203. Cambridge University Press, 1992.
- [KP08] Sebastien Konieczny and Ramón Pino Perez. Improvement operators. In *Eleventh International Conference on Principles of Knowledge Representation and Reasoning*, pages 177–186, 2008.
- [KW85] Arthur M. Keller and Marianne Winslett. On the use of an extended relational model to handle changing incomplete information. *IEEE Transactions on Software Engineering*, 11(7):620–633, 1985.
- [Leh95] Daniel J. Lehmann. Belief revision, revised. In *Proceedings of the Fourteenth International Joint Conference on Artificial Intelligence (IJCAI'95)*, pages 1534–1540, 1995.
- [Lev77] Isaac Levi. Subjunctives, dispositions, and chances. *Synthese*, 34:423–455, 1977.
- [MS88] João Martins and Stuart Shapiro. A model for belief revision. *Artificial Intelligence*, 35:25–79, 1988.
- [Nay94a] Abhaya Nayak. Foundational belief change. *Journal of Philosophical Logic*, 23:495–533, 1994.
- [Nay94b] Abhaya Nayak. Iterated belief change based on epistemic entrenchment. *Erkenntnis*, 41:353–390, 1994.
- [NPP03] Abhaya Nayak, Maurice Pagnucco, and Pavlos Peppas. Dynamic belief revision operators. *Artificial Intelligence*, 146:2,:193–228, 2003.
- [Pep14] Pavlos Peppas. A panorama of iterated revision. In Sven Ove Hansson, editor, *David Makinson on Classical Methods for Non-Classical Problems*, volume 3 of *Outstanding Contributions to Logic*, pages 71–94. Springer Netherlands, 2014.
- [Rot03] Hans Rott. Coherence and conservatism in the dynamics of belief. part ii: Iterated belief change without dispositional coherence. *Journal of Logic and Computation*, 13:111–145, 2003.
- [Rot09] Hans Rott. Shifting priorities: Simple representations for twenty-seven iterated theory change operators. In H. Wansing D. Makinson, J. Malinowski, editor, *Towards Mathematical Philosophy*, number 28 in Trends in Logic, pages 269–296. Springer Science, 2009.
- [Sat88] Ken Satoh. Nonmonotonic reasoning by minimal belief revision. In *FGCS*, pages 455–462, 1988.
- [vW71] G. H. von Wright. *Explanation and Understanding*. Cornell University Press, 1971.

- [Web86] Andreas Weber. Updating propositional formulas. In *Expert Database Conf.*, pages 487–500, 1986.
- [Win88] Marianne Winslett. Reasoning about action using a possible models approach. In *AAAI*, pages 89–93, 1988.
- [ZF96] Yan Zhang and Norman Y. Foo. Updating knowledge bases with disjunctive information. In William J. Clancey and Daniel S. Weld, editors, *AAAI/IAAI, Vol. 1*, pages 562–568. AAAI Press / The MIT Press, 1996.