# A Brief Note About Rott Contraction

EDUARDO L. FERMÉ and RICARDO O. RODRIGUEZ,
*Departamento de Computación, Universidad de Buenos Aires, Pabellón I Ciudad Universitaria (1428), Buenos Aires, Argentina.*
*E-mail: {ferme,ricardo}@dc.uba.ar*

## Abstract

One of the ways to model contraction functions for belief sets is epistemic entrenchment. The first step was provided by Gärdenfors in [5], who defined epistemic entrenchment and a contraction function in terms of it and related the latter with the AGM contraction function. Later Hans Rott in [16] presented an entrenchment based contraction function that does not satisfy recovery. In this paper we provide an axiomatic characterization of Rott Contraction.[1]

*Keywords*: AGM, belief contraction, recovery postulate, Rott contraction.

## 1 Introduction

*"Even if all sentences in a knowledge set are accepted or considered as facts, this does not means that all sentences are of equal value for planning or problem solving purposes. Certain pieces of knowledge and belief about the world are more important than others when planning future actions, conducting scientific investigations or reasoning in general. We will say that some sentences in a knowledge system have a higher degree of epistemic entrenchment than others. The degree of entrenchment will, intuitively, have a bearing on what is abandoned from a knowledge set and what is retained, when a contraction or revision is carried out."* [6]

This is the key idea of epistemic entrenchment introduced by Gärdenfors in [5]. Using epistemic entrenchment, Gärdenfors defines an operation of contraction that satisfies the entire set of AGM contraction postulates [5, 6].
However, epistemic entrenchment also allows us to define *entrenchment based contraction functions* that do not satisfy all the AGM postulates. One of the more important such functions in the literature was proposed by Hans Rott in [16] and has been called Rott contraction. This contraction differs from the Gärdenfors contraction in not satisfying the controversial postulate of recovery. For discussions of recovery see [3, 12, 9, 10, 11, 7, 15, 14, 13]. In [11] Lindström and Rabinowicz suggest that any realistic entrenchment-based contraction operator should lie between those of Rott and Gärdenfors.

[1]Full version of a contributed paper presented at the *4th Workshop on Logic, Language, Information and Computation* (*WoLLIC'97*), http://www.di.ufpe.br/~wollic97, held in Fortaleza (Ceará), Brazil, August 19–22 1997, with scientific sponsorship by IGPL, FoLLI and ASL, and organised by Univ. Federal do Ceará (UFC) and Univ. Federal de Pernambuco (UFPE).

 

We give here an axiomatic characterization for Rott's entrenchment-based contraction operator, i.e. a characterization of the lower limit of what Lindström and Rabinowicz regard as reasonable contraction.

In Section 2 we recall *AGM contraction functions*, introduce epistemic entrenchment, relate it to AGM contraction, and also present Rott contraction. In Section 3 we provide the promised axiomatic characterization of Rott contraction. Proofs are in the appendix.

After this work was finished we were informed that another axiomatic characterization has been independently obtained by Hans Rott and Maurice Pagnucco and will be published in a joint paper by these authors [17].

## 2 Background

### *2.1 AGM contraction*

The logic of theory change was introduced into philosophical logic and artificial intelligence a little over a decade ago. The initial step was provided by Alchourrón, Gärdenfors and Makinson in [1] (commonly called the AGM model). Basically, in the AGM model the beliefs of a rational agent are represented by a belief set $\mathbf{K}$, closed under logical consequence $Cn$, where $Cn$ satisfies the Tarski conditions $\mathbf{K} \subseteq Cn(\mathbf{K})$ for any set $\mathbf{K}$ of propositions, $Cn(Cn(\mathbf{K})) \subseteq Cn(\mathbf{K})$ and $Cn(\mathbf{K}) \subseteq Cn(\mathbf{H})$ if $\mathbf{K} \subseteq \mathbf{H}$ . We assume that $Cn$ includes classical logical consequence, satisfies the rule of introduction of disjunction into premises and is compact. We write $\vdash x$ for $x \in \mathrm{Cn}(\emptyset)$.

A theory is understood to be any set $\mathbf{K}$ of proposition closed under $Cn$, i.e. such that $Cn(\mathbf{K}) = \mathbf{K}$.

Let $\mathbf{L}$ be the set of all the sentences of the language. Let $\mathcal{K}$ the set of all theories of the language. The expansion function $+$ from $\mathcal{K} \times \mathbf{L}$ to $\mathcal{K}$ is defined by $(\mathbf{K} + x) = Cn(\mathbf{K} \cup \{x\})$. The postulates for the AGM contraction are:

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|---|---|---|
| $(\mathbf{K-1})$ | $\mathbf{K} - x$ is a theory whenever $\mathbf{K}$ is a theory | (closure) |
| $(\mathbf{K-2})$ | $\mathbf{K} - x \subseteq \mathbf{K}$ | (inclusion) |
| $(\mathbf{K-3})$ | If $x \notin \mathbf{K}$, then $\mathbf{K} - x = \mathbf{K}$ | (vacuity) |
| $(\mathbf{K-4})$ | If $x \notin Cn(\emptyset)$ , then $x \notin \mathbf{K} - x$ | (success) |
| $(\mathbf{K-5})$ | If $x \leftrightarrow y \in Cn(\emptyset)$ then $\mathbf{K} - x = \mathbf{K} - y$ | (extensionality) |
| $(\mathbf{K-6})$ | $\mathbf{K} \subseteq (\mathbf{K} - x) + x$ whenever $\mathbf{K}$ is a theory | (recovery) |
| $(\mathbf{K-7})$ | $\mathbf{K} - x \cap \mathbf{K} - y \subseteq \mathbf{K} - (x \wedge y)$ | (conjunctive overlap) |
| $(\mathbf{K-8})$ | If $x \notin \mathbf{K} - (x \wedge y)$, then $\mathbf{K} - (x \wedge y) \subseteq \mathbf{K} - x$ | (conjunctive inclusion). |

### *2.2 Epistemic entrenchment*

The idea of entrenchment for theories was introduced by Gärdenfors in [5] to represent formally a preference ordering between formulae in a theory. He attempted to define the contraction of a theory by a sentence in terms of an order of the sentences, and identify the properties that the order must satisfy for the generated contraction to satisfy the AGM postulates.

Gärdenfors proposed the following set of axioms for the order among sentences:

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|---|---|---|
| (**EE1**) | If $x \leq_{\mathbf{K}} y$ and $y \leq_{\mathbf{K}} z$, then $x \leq_{\mathbf{K}} z$ | (transitivity) |
| (**EE2**) | If $x \vdash y$, then $x \leq_{\mathbf{K}} y$ | (dominance) |
| (**EE3**) | $x \leq_{\mathbf{K}} (x \wedge y)$ or $y \leq_{\mathbf{K}} (x \wedge y)$ | (conjunctiveness) |
| (**EE4**) | If $\mathbf{K} \neq \mathbf{K}_{\perp}$, then $x \notin \mathbf{K}$ if and only if $x \leq_{\mathbf{K}} y$ for all $y$ | (minimality) |
| (**EE5**) | If $y \leq_{\mathbf{K}} x$ for all $y$, then $\vdash x$ | (maximality) |

A relation satisfying (**EE1**) – (**EE5**) is a *standard entrenchment ordering*. Furthermore, he showed that entrenchment orderings can be connected with contraction functions by the following equivalences [2]:

$(C \leq)$ $\quad x \leq_{\mathbf{K}} y$ if and only if $x \notin \mathbf{K}-(x \wedge y)$ or $\vdash (x \wedge y)$.

Gärdenfors' entrenchment-based contraction
$(-_G)$ $\quad y \in \mathbf{K}-x$ if and only if $y \in \mathbf{K}$ and, either $\vdash x$ or $x <_{\mathbf{K}} (x \vee y)$.

**Observation 2.1 ([5, 6])** *Let $\leq_{\mathbf{K}}$ be a standard entrenchment ordering on a consistent belief set $\mathbf{K}$. Furthermore let $-_G$ be the Gärdenfors entrenchment-based contraction on $\mathbf{K}$ defined by condition $(-_G)$ from $\leq_{\mathbf{K}}$. Then $-_G$ satisfies the eight AGM postulates, and $(C \leq)$ also holds.*

**Observation 2.2 ([5, 6])** *Let $-$ be an operation on a consistent belief set $\mathbf{K}$ that satisfies the eight AGM postulates. Furthermore let $\leq_{\mathbf{K}}$ be the relation defined from $-$ by condition $(C \leq)$. Then $\leq_{\mathbf{K}}$ satisfies the standard entrenchment postulates and $(-_G)$ also holds.*

Hans Rott [16] has remarked that the comparison $x <_{\mathbf{K}} (x \vee y)$ is not intuitive, and proposed the following alternative definition of a contraction operation from an entrenchment ordering:

Rott's entrenchment-based contraction
$(-_R)$ $\quad y \in \mathbf{K}-x$ if and only if $y \in \mathbf{K}$ and, either $\vdash x$ or $x <_{\mathbf{K}} y$.

Rott also provided the following result:

**Observation 2.3** *Let $\leq_{\mathbf{K}}$ be a standard entrenchment ordering on a consistent belief set $\mathbf{K}$. Furthermore let $-_R$ be the Rott entrenchment-based contraction on $\mathbf{K}$ defined by condition $(-_R)$ from $\leq_{\mathbf{K}}$. Then $-_R$ satisfies all the AGM postulates except* recovery.

Rott [16, p. 169] proved that for all $x$, $\mathbf{K}-_{\mathrm{R}}x \subseteq \mathbf{K}-_{\mathrm{G}}x$. Lindström and Rabinowicz [11] have proposed that a *reasonable* entrenchment based contraction operation $-$ should lie between Rott's operation and Gärdenfors' operation, in the sense that $\mathbf{K} -_{\mathrm{R}} x \subseteq \mathbf{K} - x \subseteq \mathbf{K} -_{\mathrm{G}} x$.

[2] We write $x <_{\mathbf{K}} y$ to denote $x \leq_{\mathbf{K}} y$ and $y \not\leq_{\mathbf{K}} x$.

## 3 An axiomatic characterization of Rott contraction

In section 2.2 we recalled that Hans Rott proved that his contraction satisfies all the AGM postulates except *recovery*. In this section we provide it with an axiomatic characterization. We make use of the following postulates:

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| If $\mathbf{K} - (x \wedge y) \subseteq \mathbf{K} - y$ then $y \notin \mathbf{K} - x$ or $\vdash x$ or $\vdash y$ | (Converse Conjunctive Inclusion). |
| If $x \in Cn(\emptyset)$ then $\mathbf{K} - x = \mathbf{K}$ | (Failure) [4]. |
| If $x \notin \mathbf{K} - y$ then $\mathbf{K} - y \subseteq \mathbf{K} - x$ | (Strong Inclusion) |

Now we can characterize Rott Contraction in terms of postulates:

**Theorem 3.1**

1. *Let $\leq_{\mathbf{K}}$ be a standard entrenchment ordering on a consistent belief set $\mathbf{K}$. Furthermore, let $-_R$ be Rott's entrenchment-based contraction on $\mathbf{K}$, defined from $\leq_{\mathbf{K}}$ by condition $(-_{\mathrm{R}})$. Then $-_R$ satisfies* closure, inclusion, success, extensionality, conjunctive overlap, failure, strong inclusion *and* converse conjunctive inclusion, *and $(C \leq)$ also holds.*

2. *Let $-$ be an operation on a consistent belief set $\mathbf{K}$ that satisfies* closure, inclusion, success, extensionality, conjunctive overlap, failure, strong inclusion *and* converse conjunctive inclusion. *Furthermore let $\leq_{\mathbf{K}}$ be the relation that is defined from $-$ by $(C \leq)$. Then $\leq_{\mathbf{K}}$ satisfies the standard entrenchment postulates, and $(-_R)$ also holds.*

Other interesting postulates are:

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| If $\nvdash x$ and $\nvdash y$, then either $x \notin \mathbf{K} - y$ or $y \notin \mathbf{K} - x$ | (Expulsiveness)[8]. |
| $\mathbf{K} - (x \wedge y) = \mathbf{K} - x$ or $\mathbf{K} - (x \wedge y) = \mathbf{K} - y$ | (Linear Hierarchical Ordering) |
| $\mathbf{K} - y \subseteq \mathbf{K} - x$ or $\mathbf{K} - x \subseteq \mathbf{K} - y$ | (Linearity) |

The following relations hold:

**Observation 3.2** *Let $\mathbf{K}$ be a belief set and $-$ an operator from $\mathcal{K} \times \mathbf{L}$ to $\mathcal{K}$. Then:*

1. *If $-$ satisfies* strong inclusion *then it satisfies* conjunctive inclusion.

2. *If $-$ satisfies* inclusion, failure *and* strong inclusion *then it satisfies* vacuity.

3. *If $-$ satisfies* closure, success *and* strong inclusion *then it satisfies* expulsiveness.

4. *If* $-$ *satisfies* inclusion, failure, strong inclusion *and* expulsiveness *then it satisfies* linearity*.*

5. *If* $-$ *satisfies* closure, success, extensionality *and* strong inclusion *then it satisfies* linear hierarchical ordering*.*

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## Appendix: Proofs

***Proof of Theorem 3.1***
**Part 1:** *Closure, inclusion, success, extensionality* and *conjunctive overlap* follow from observation 2.3.

**Converse conjunctive inclusion**: Let $\mathbf{K}-(x \wedge y) \subseteq \mathbf{K}-y$. We have two cases: 1. $y \in \mathbf{K}-y$: It follows from *success* that $\vdash y$. 2. $y \notin \mathbf{K}-y$: then $y \notin \mathbf{K}-(x \wedge y)$. It follows from $(-_R)$ that $y \notin \mathbf{K}$ or ($y \leq_{\mathbf{K}} x \wedge y$ and $\nvdash x \wedge y$). We have two subcases 2.1. $y \notin \mathbf{K}$: then by *inclusion* (see observation 2.3) $y \notin \mathbf{K}-x$. 2.2. $y \in \mathbf{K}$: then $y \leq_{\mathbf{K}} x \wedge y$. By (**EE2**) $x \wedge y \leq_{\mathbf{K}} x$ and by (**EE1**) $y \leq_{\mathbf{K}} x$ hence by $(-_R)$ $y \notin \mathbf{K}-x$ or $\vdash x$ .

**Failure**: It follows trivially, since if $\vdash x$, then by $(-_R)$ $y \in \mathbf{K}-x$ if and only if $y \in \mathbf{K}$.

**Strong inclusion**: Let $x \notin \mathbf{K}-y$. By $(-_R)$ $x \notin \mathbf{K}$ or ($\nvdash y$ and $x \leq_{\mathbf{K}} y$). We have two subcases: 1. $x \notin \mathbf{K}$: then $\mathbf{K}-x = \mathbf{K}$ (since by observation 2.3, $-$ satisfies *vacuity*); then $\mathbf{K}-y \subseteq \mathbf{K}-x$ (since by observation 2.3, $-$ satisfies *inclusion*). 2. $x \in \mathbf{K}$: then ($\nvdash y$ and $x \leq_{\mathbf{K}} y$) Let $z \in \mathbf{K}-y$, then (by $(-_R)$) $z \in \mathbf{K}$ and $y <_{\mathbf{K}} z$. By (**EE1**) $x <_{\mathbf{K}} z$ then by $(-_R)$ $z \in \mathbf{K}-x$ hence $\mathbf{K}-y \subseteq \mathbf{K}-x$.

$(C\ \leq)$[3]**:** For one direction let $x \leq_{\mathbf{K}} y$ and $x \in \mathbf{K}-(x \wedge y)$. We need to prove $\vdash x \wedge y$. By $(-_R)$ we have: $x \in \mathbf{K}-(x \wedge y)$ if and only if $x \in \mathbf{K}$ and either $\vdash x \wedge y$ or $x \wedge y <_{\mathbf{K}} x$. Therefore: $\vdash x \wedge y$ or $x \wedge y <_{\mathbf{K}} x$. Let $x \wedge y <_{\mathbf{K}} x$: then, by (**EE3**), $y \leq_{\mathbf{K}} x \wedge y$; and since $x \leq_{\mathbf{K}} y$, we have by (**EE1**) that $x \leq_{\mathbf{K}} (x\wedge \text{y})$, contradiction, hence $\vdash x \wedge y$.
For the second direction we have two subcases: 1. $x \notin \mathbf{K}-(x \wedge y)$: Then by $(-_R)$, $x \notin \mathbf{K}$ or $\nvdash x$ and $x \leq_{\mathbf{K}} x \wedge y$. If $x \notin \mathbf{K}$, $x \leq_{\mathbf{K}} y$ follows (by (**EE4**)). If $x \leq_{\mathbf{K}} x \wedge y$, by (**EE1**), (since by (**EE2**) $x \wedge y \leq_{\mathbf{K}} y$), $x \leq_{\mathbf{K}} y$. 2. $\vdash x \wedge y$, then $\vdash y$, hence by (**EE2**), $x \leq_{\mathbf{K}} y$. This completes the proof.

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[3]The idea for this proof was provided by an anonymous referee.

Part 2: (**EE2**) – (**EE5**) are proved by Gärdenfors and Makinson from *closure, inclusion, success, extensionality, failure* and $(C \leq)$ in [6], pp. 93-94.

• (**EE1**)

We demonstrate by *reductio ad absurdum*. Let $x \leq_{\mathbf{K}} y$, $y \leq_{\mathbf{K}} z$ and $x \not\leq_{\mathbf{K}} z$. It follows by $(C \leq)$ that: (a) either $\vdash x \wedge y$ or $x \notin \mathbf{K}-(x \wedge y)$; (b) either $\vdash y \wedge z$ or $y \notin \mathbf{K}-(y \wedge z)$; and (c) $\nvdash x \wedge z$ and $x \in \mathbf{K}-(x \wedge z)$.
1. Let $\vdash x \wedge y$: then $\vdash x$ and $\vdash y$. By *closure* $y \in \mathbf{K}-(y \wedge z)$, then by condition (b) $\vdash y \wedge z$, so $\vdash z$, and $\vdash x \wedge z$; contradiction.
2. Let $\vdash y \wedge z$: then $\vdash y$ and $\vdash z$. By *closure* $z \in \mathbf{K}-(x \wedge z)$, then by condition (c) and *closure* $x \wedge z \in \mathbf{K}-(x \wedge z)$; hence by *success* $\vdash x \wedge z$; contradiction.
By 1. and 2. (a), (b) and (c) are reduced to $x \notin \mathbf{K}-(x \wedge y)$, $y \notin \mathbf{K}-(y \wedge z)$, $\nvdash x \wedge z$ and $x \in \mathbf{K}-(x \wedge z)$.
3. By *strong inclusion* $\mathbf{K}-(x \wedge y) \subseteq \mathbf{K}-x$, then by *converse conjunctive inclusion* we have $\vdash x$ or $\vdash y$ or $x \notin \mathbf{K}-y$. But since $x \notin \mathbf{K}-(x \wedge y)$ and $y \notin \mathbf{K}-(y \wedge z)$, by *closure* $\nvdash x$ and $\nvdash y$. Then $x \notin \mathbf{K}-y$ and by *strong inclusion* we have $\mathbf{K}-y \subseteq \mathbf{K}-x$.
4. $y \notin \mathbf{K}-(y \wedge z)$ implies by *strong inclusion* that $\mathbf{K}-(y \wedge z) \subseteq \mathbf{K}-y$, then by *converse conjunctive inclusion* we have $\vdash y$ or $\vdash z$ or $y \notin \mathbf{K}-z$; by *closure* $\nvdash y$ and $\nvdash z$ (since by *success, closure* and condition **(C)**, $z \notin \mathbf{K}-(x \wedge z)$). Then $y \notin \mathbf{K}-z$ and by *strong inclusion* we have $\mathbf{K}-z \subseteq \mathbf{K}-y$.
5. It follows from *success* that $z \notin \mathbf{K}-z$, so by *closure* $(x \wedge z) \notin \mathbf{K}-z$; and since $z \notin \mathbf{K}-(x \wedge z)$ we obtain by *strong inclusion* that $\mathbf{K}-z = \mathbf{K}-(x \wedge z)$. So $x \in \mathbf{K}-(x \wedge z) = \mathbf{K}-z \subseteq \mathbf{K}-y \subseteq \mathbf{K}-x$. Hence by *success* $\vdash x$; contradiction.

• $-_R$

$\Rightarrow$)

Let $y \in \mathbf{K}-x$ and $\nvdash x$. It follows by *inclusion* that $y \in \mathbf{K}$. We have two cases: 1. $\vdash y$: By *closure* $y \in \mathbf{K}-(x \wedge y)$, then by $(C \leq)$, $y \not\leq_{\mathbf{K}} x$. By *success* and *closure* $x \notin \mathbf{K}-(x \wedge y)$; then, by $(C \leq)$, $x \leq_{\mathbf{K}} y$. Hence $x <_{\mathbf{K}} y$.
2. $\nvdash y$: By *converse conjunctive inclusion* $\mathbf{K}-(x \wedge y) \nsubseteq \mathbf{K}-y$, then by *strong inclusion* $y \in \mathbf{K}-(x \wedge y)$; then by $(C \leq)$, $y \not\leq_{\mathbf{K}} x$. By *success* and *closure* $x \notin \mathbf{K}-(x \wedge y)$; then, by $(C \leq)$, $x \leq_{\mathbf{K}} y$. Hence $x <_{\mathbf{K}} y$.

$\Leftarrow$)

1. Let $y \in \mathbf{K}$ and $\vdash x$. By *failure* $\mathbf{K}-x = \mathbf{K}$ then $y \in \mathbf{K}-x$.
2. Let $y \in \mathbf{K}$ and $x <_{\mathbf{K}} y$. By $(C \leq)$ $x \notin \mathbf{K}-(x \wedge y)$, then by *strong inclusion* $\mathbf{K}-(x \wedge y)$. For *reductio ad absurdum* let $y \notin \mathbf{K}-x$; then $y \notin \mathbf{K}-(x \wedge y)$ then by $(C \leq)$, $y \leq_{\mathbf{K}} x$. Contradiction.

***Proof of Observation 3.2***

1. If $-$ satisfies *strong inclusion* then it satisfies *conjunctive inclusion*:
   Let $x \notin \mathbf{K}-(x \wedge y)$. Then by *strong inclusion* $\mathbf{K}-(x \wedge y) \subseteq \mathbf{K}-x$.

2. If $-$ satisfies *inclusion, failure* and *strong inclusion* then it satisfies *vacuity*:
   Let $x \notin \mathbf{K}$ and $\vdash y$. Then by *failure* $x \notin \mathbf{K}-y = \mathbf{K}$; by *strong inclusion* $\mathbf{K} = \mathbf{K}-y \subseteq \mathbf{K}-x$. Hence by *inclusion* $\mathbf{K}-x = \mathbf{K}$.

3. If $-$ satisfies *closure, success* and *strong inclusion* then it satisfies *expulsiveness*:
   Let $\nvdash x$ and $\nvdash y$. By *closure* and *success* $x \wedge y \notin \mathbf{K}-x$ and $x \wedge y \notin \mathbf{K}-y$, then by *strong inclusion* $\mathbf{K}-x \subseteq \mathbf{K}-(x \wedge y)$ and $\mathbf{K}-y \subseteq \mathbf{K}-(x \wedge y)$. Let $y \in \mathbf{K}-x$, then $y \in \mathbf{K}-(x \wedge y)$, then by *success* $x \notin \mathbf{K}-(x \wedge y)$; hence $x \notin \mathbf{K}-y$.

4. If $-$ satisfies *inclusion, failure, strong inclusion* and *expulsiveness* then it satisfies *linearity*:
   If $\vdash x$ then by *failure* $\mathbf{K}-x = \mathbf{K}$, and by *inclusion* $\mathbf{K}-y \subseteq \mathbf{K}-x$. By the same reasoning if $\vdash y$ then $\mathbf{K}-x \subseteq \mathbf{K}-y$. Let $\nvdash x$ and $\nvdash y$, then by *expulsiveness* $y \notin \mathbf{K}-x$ or $x \notin \mathbf{K}-y$. Hence by *strong inclusion* $\mathbf{K}-x \subseteq \mathbf{K}-y$ or $\mathbf{K}-y \subseteq \mathbf{K}-x$.

5. If $-$ satisfies *closure, success, extensionality* and *strong inclusion* then it satisfies *linear hierarchical ordering*:
   If $\vdash x$ or $\vdash y$ then $\vdash (x \wedge y) \leftrightarrow x$ or $\vdash (x \wedge y \leftrightarrow y)$, then by *extensionality* $\mathbf{K}-(x \wedge y) = \mathbf{K}-x$ or $\mathbf{K}-(x \wedge y) = \mathbf{K}-y$. Let $\nvdash x$ and $\nvdash y$. By *closure* and *success* $x \wedge y \notin \mathbf{K}-x$ and $x \wedge y \notin \mathbf{K}-y$, then by *strong inclusion* $\mathbf{K}-x \subseteq \mathbf{K}-(x \wedge y)$ and $\mathbf{K}-y \subseteq \mathbf{K}-(x \wedge y)$. By *success* $x \notin \mathbf{K}-(x \wedge y)$ or $y \notin \mathbf{K}-(x \wedge y)$. Then by *strong inclusion* $\mathbf{K}-x \subseteq \mathbf{K}-(x \wedge y)$ or $\mathbf{K}-y \subseteq \mathbf{K}-(x \wedge y)$ Hence $\mathbf{K}-(x \wedge y) = \mathbf{K}-x$ or $\mathbf{K}-(x \wedge y) = \mathbf{K}-y$.

## References


[1] Carlos Alchourrón, Peter Gärdenfors, and David Makinson. On the logic of theory change: Partial meet contraction and revision functions. *Journal of Symbolic Logic*, 50:510–530, 1985.

[2] Carlos Alchourrón and David Makinson. On the logic of theory change: Contraction functions and their associated revision functions. *Theoria*, 48:14–37, 1982.

[3] Eduardo Fermé. Five faces of recovery. (manuscript), 1998.

[4] André Fuhrmann and Sven Ove Hansson. A survey of multiple contraction. *Journal of Logic, Language and Information*, 3:39–76, 1994.

[5] Peter Gärdenfors. *Knowledge in Flux: Modeling the Dynamics of Epistemic States.* MIT Press, Cambridge, 1988.

[6] Peter Gärdenfors and David Makinson. Revisions of knowledge systems using epistemic entrenchment. In Moshe Y. Vardi, editor, *Proceedings of the Second Conference on Theoretical Aspects of Reasoning About Knowledge*, pages 83–95, Monterey, California, March 1988. Morgan Kaufmann.

[7] Sven Ove Hansson. Belief contraction without recovery. *Studia Logica*, 50:251–260, 1991.

[8] Sven Ove Hansson. *A Textbook of Belief Dynamics.* Kluwer Academic Publisher, Dordrecht, In Press.

[9] Isaac Levi. *The fixation of belief and its undoing: changing beliefs through inquiry.* Cambridge University Press, Cambridge, 1991.

[10] Isaac Levi. Contraction and informational value. (manuscript, fifth version), August 1997.

[11] Sten Lindström and Wlodzimierz Rabinowicz. Epistemic entrenchment with incomparabilities and relational belief revision. In Fuhrmann and Morreau, editors, *The Logic of Theory Change*, pages 93–126, Berlin, 1991. Springer-Verlag.

[12] David Makinson. On the status of the postulate of recovery in the logic of theory change. *The Journal of Philosophical Logic*, 16:383–394, 1987.

[13] David Makinson. On the force of some apparent counterexamples to recovery. In E. Garzón Valdéz et all, editor, *Normative Systems in Legal and Moral Theory: Festschrift for Carlos Alchourrón and Eugenio Bulygin*, pages 475–481, Berlin, 1997. Duncker & Humblot.

[14] Abhaya Nayak. Foundational belief change. *Journal of Philosophical Logic*, 23:495–533, 1994.

[15] Reinhard Niederée. Multiple contraction. a further case against Gärdenfors' principle of recovery. In Fuhrmann and Morreau, editors, *The Logic of Theory Change*, pages 322–334, Berlin, 1991. Springer-Verlag.

[16] Hans Rott. Two methods of constructing contractions and revisions of knowledge systems. *Journal of Philosophical Logic*, 20:149–173, 1991.

[17] Hans Rott and Maurice Pagnucco. Severe withdrawal (and recovery). (manuscript), 1997.

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