Plasma Sources Science and Technology

PAPER • OPEN ACCESS

# Ionization layer with collision-free atoms at the edge of partially to fully ionized plasmas

To cite this article: M S Benilov 2024 *Plasma Sources Sci. Technol.* **33** 055002

OPEN ACCESS
IOP Publishing
Plasma Sources Science and Technology
Plasma Sources Sci. Technol. **33** (2024) 055002 (8pp)
https://doi.org/10.1088/1361-6595/ad3f49

# Ionization layer with collision-free atoms at the edge of partially to fully ionized plasmas

**M S Benilov**

Departamento de Física, Faculdade de Ciências Exatas e da Engenharia, Universidade da Madeira, 9000 Funchal, Portugal
Instituto de Plasmas e Fusão Nuclear, Instituto Superior Técnico, Universidade de Lisboa, 1049-001 Lisbon, Portugal

E-mail: benilov@staff.uma.pt

Received 15 December 2023, revised 6 March 2024
Accepted for publication 16 April 2024
Published 7 May 2024

**Abstract**

When a hot arc spot has just formed on the cathode surface, e.g. in the course of arc ignition on a cold cathode, a significant part of the current still flows in the glow-discharge mode to the cold surface outside the spot. The near-cathode voltage continues to be high at all points of the cathode surface. The mean free path for collisions between the atoms and the ions within the plasma ball near the spot is comparable to, or exceeds, the thickness of the ionization layer, which is a part of the near-cathode non-equilibrium layer where the ion current to the cathode is generated. The evaluation of the ion current to the cathode surface under such conditions is revisited. A fluid description of the ion motion in the ionization layer is combined with a kinetic description of the atom motion. The resulting problem admits a simple analytical solution. Formulas for the evaluation of the ion current to the cathode for a wide range of conditions are derived and the possibilities of using these formulas to improve the accuracy of existing methods for modeling high-pressure arc discharges in relation to glow-to-arc transitions are discussed.

Keywords: ionization layer, arc discharges, modelling of arc discharges, near-cathode phenomena

## 1. Introduction

Ionization (Saha) equilibrium, which holds in partially to fully ionized high-pressure plasmas, e.g. those generated in high-current arc discharges, is violated in thin layers near solid surfaces contacting the plasma. Of particular importance is the non-equilibrium layer near the cathode, since it is in this layer that the ion current to the cathode surface is formed, which heats the surface to the high temperatures necessary for electron emission. More precisely, the ion current is formed in the outer–quasi-neutral–section of the near-cathode non-equilibrium layer; the so-called ionization layer. An understanding and adequate theoretical description of the ionization layer are needed for evaluation of the ion current to the cathode.

The physics of the ionization layer is relatively well known (e.g. review [1] and references therein) and may be briefly described as follows. Ions generated in the ionization layer move to the cathode surface where they recombine. (On their way to the cathode, the ions cross the space-charge sheath, where they are accelerated by the sheath electric field, but this is not directly relevant to this context.) Neutral atoms thus formed are desorbed from the surface and move into the plasma, with some or all of them being ionized upon reaching the ionization layer. A theoretical description of the relative motion of the ions and the atoms in the ionization layer depends on the relationship between the scale of thickness of the ionization layer, $l$, and the mean free path for collisions between the atoms and the ions, $\lambda_{ia}$.

The physics of the ionization layer is considered in a number of works [2–6]. In the fully developed arc-cathode regime, the current-collecting part of the cathode surface is hot and the near-cathode voltage drop is low. $l > \lambda_{ia}$ under such conditions and the ionization layer may be described in the diffusion approximation. On the other hand, estimates show that situations where $l \lesssim \lambda_{ia}$ occur during glow-to-arc transitions, including those that take place in the course of arc ignition on cold cathodes: when a hot arc spot has just formed on the cathode surface, a significant part of current still flows to the cold surface outside the spot in the glow-discharge regime, and therefore the near-cathode voltage continues to be high at all points of the cathode surface including in the spot. The coupling between the ion and atom species in the ionization layer is not strong in such situations and the diffusion description of the ion-atom relative motion in the layer is not valid.

In [2–6], a theory of the ionization layer for the case $l \lesssim \lambda_{ia}$ was developed in the framework of the multifluid approximation, which considers ions and atoms as separate fluids coexisting with each other. In this work, the theory is revisited. Since the mean free path for ion-ion collisions is small compared to the thickness of the ionization layer while the mean free path for atom-atom collisions is large, a fluid description of the ion motion, similar to the one in [2–6], is combined with a kinetic description of the motion of the atoms. The resulting problem admits a simple analytical solution, which is used to derive formulas for evaluation of the ion current to the cathode surface for a wide range of conditions. The possibilities of using the obtained results to improve the accuracy of existing methods for modeling high-pressure arc discharges (e.g. review [7]; see also [8–15] as further examples) in relation to glow-to-arc transitions are discussed.

The outline of the paper is as follows. Estimates of characteristic length scales using the example of atmospheric-pressure argon plasma are given in section 2. Theory of ionization layer for the limiting case $l \ll \lambda_{ia}$ is developed in section 3. In section 4, formulas for the ion current to the cathode surface for a wide range of conditions are derived. Summary and concluding remarks are given in section 5.

## 2. Characteristic length scales

Let us consider the ionization layer in a high-pressure arc plasma, comprising neutral ions, singly charged positive ions, and the electrons. The presence of multiply charged ions is neglected: numerical calculations of the ionization layer in 1 atm Ar plasma with account of atoms, electrons, singly, doubly, and triply charged ions [16] have shown that the dominant type of ions in the near-surface region is $Ar^+$ for electron temperatures at least up to 50 000 K, which is a consequence of the decrease of the rate constant of each subsequent ionization. The distribution of plasma electrons is Maxwellian with a temperature $T_\mathrm{e}$, the density of non-thermalized (just emitted) electrons is negligible. $T_\mathrm{e}$ is constant across the ionization layer; a usual assumption of approximate models of near-cathode layers in high-pressure arc plasmas supported by numerical modelling (e.g. [17]). The heavy-particle temperature $T_\mathrm{h}$ is constant across the ionization layer as well and coincides with $T_\mathrm{c}$ the temperature of the cathode surface.

Let us designate by $\bar{Q}_{ii}^{(1,1)}$, $\bar{Q}_{aa}^{(1,1)}$, and $\bar{Q}_{ia}^{(1,1)}$ energy-averaged cross sections for momentum transfer in ion-ion, atom-atom, and ion-atom collisions. These quantities are functions of the heavy-particle temperature $T_\mathrm{h}$ and are constant across the ionization layer. Let us define characteristic mean free paths for collisions between the ions, between the neutral atoms, and between the neutral atoms and the ions,

$$\lambda_{ii} = \frac{1}{n_i^{(0)}\bar{Q}_{ii}^{(1,1)}}, \quad \lambda_{aa} = \frac{1}{n_a^{(0)}\bar{Q}_{aa}^{(1,1)}}, \quad \lambda_{ia} = \frac{1}{n_h^{(0)}\bar{Q}_{ia}^{(1,1)}}, \quad (1)$$

where $n_i$ and $n_a$ are the number densities of ion and atoms; $n_h = n_a + n_i$ is the number density of heavy particles; and the upper index $(0)$ denotes characteristic values in the ionization layer. Note that $\lambda_{ia}$ defined in this way represents the mean free path of an ion in the gas of atoms in the case of weakly ionized plasma, $n_i^{(0)} \ll n_a^{(0)}$, and the mean free path of an atom in the gas of ions in the case of plasma close to full ionization, $n_a^{(0)} \ll n_i^{(0)}$. Let us assume for definiteness that $n_i^{(0)}$, $n_a^{(0)}$, and $n_h^{(0)}$ are estimated at the 'edge' of the ionization layer, where the plasma is in ionization equilibrium and the Saha equation applies, then these values may be evaluated in terms of $T_\mathrm{e}$, $T_\mathrm{h}$, and the plasma pressure.

The dominant mechanism of ionization in atomic plasmas is ionization of neutral atoms by electron impact. A characteristic time of ionization of an atom may be estimated as $(k_i n_e^{(0)})^{-1}$, where $k_i$ is the rate constant of ionization of neutral atoms by electron impact and $n_e$ is the electron number density (we set $n_e^{(0)} = n_i^{(0)}$). $k_i$ is a function of the electron temperature $T_\mathrm{e}$ and is constant across the ionization layer. The scale of thickness of the ionization layer, $l$, may be evaluated as an average distance from the surface on which a desorbed neutral atom gets ionized:

$$l = \frac{v_a^{(0)}}{k_i n_e^{(0)}}, \quad (2)$$

where $v_a^{(0)}$ is a characteristic speed with which the atoms move from the cathode into the plasma.

$v_a^{(0)}$ depends on the character of the motion of an average atom before it gets ionized. In the limiting case where the atom moves without collisions, $v_a^{(0)}$ may be estimated as the average normal velocity with which the atoms are desorbed from the cathode, $v_a^{(0)} = \sqrt{2kT_\mathrm{c}/\pi m_a}$, where $m_a$ is the atom mass. Designating the scale of thickness of the ionization layer in this limiting case by $l_{\rm cf}$, where cf stands for collision-free, and replacing $T_{\rm c}$ with $T_{\rm h}$, one obtains

$$l_{\rm cf} = \sqrt{\frac{2kT_{\rm h}}{\pi m_a}} \frac{1}{k_i n_e^{(0)}}. \quad (3)$$

The condition of occurrence of the regime of collision-free motion of the atoms may be written as $l_{\rm cf} \ll \lambda_{aa}, \lambda_{ia}$. Given that the cross section $\bar{Q}_{ia}^{(1,1)}$, which is governed by the resonance charge exchange, exceeds $\bar{Q}_{aa}^{(1,1)}$ under conditions of interest, one can conclude that $\lambda_{ia} < \lambda_{aa}$ and rewrite the condition of occurrence of the regime of collision-free atoms in the form $l_{\rm cf} \ll \lambda_{ia}$.

In the opposite limiting case, where the atom-ion collisions in the ionization layer are frequent, the relative motion of the ions and the atoms may be described in the diffusion approximation. Let us designate the scale of thickness of the ionization layer in this limiting case by $l_{\rm dif}$. The characteristic atomic speed $v_a^{(0)}$ may be set equal to a characteristic speed of diffusion of the atoms,

$$v_a^{(0)} = \frac{\tilde{D}_{ia}}{l_{\rm dif}}, \quad \tilde{D}_{ia} = \frac{3\pi}{8}\sqrt{\frac{kT_{\rm h}}{\pi m_a}}\lambda_{ia}. \quad (4)$$

Note that $\tilde{D}_{ia}$ has the meaning of the binary-diffusion coefficient for a mixture constituted by the ion and neutral species, evaluated in the first approximation in expansion in Sonine polynomials in the method of Chapman–Enskog; see [18–20].

Using equation (2), one obtains

$$l_{\rm dif} = \left[\frac{\tilde{D}_{ia}}{k_i n_e^{(0)}}\right]^{1/2}. \quad (5)$$

The motion of the atoms is dominated by collisions provided that $l_{\rm dif} \gg \lambda_{ia}$.

The length scales (1), (3), and (5) are shown in figure 1 for 1 atm Ar plasma and $T_{\rm h} = 3000$ K. Also shown are the ionization degree $\omega = n_i^{(0)}/n_h^{(0)}$, the Debye length $\lambda_{\rm D}$, and the parameter $\alpha$, which characterizes the ratio of the scale of thickness of the ionization layer to the mean free path for collisions between the atoms and the ions and is defined as (cf equation (56) of [3])

$$\alpha = \left[\frac{8}{3}\frac{\bar{Q}_{ia}^{(1,1)}}{k_i}\sqrt{\frac{kT_{\rm h}}{\pi m_a}}\right]^{1/2}. \quad (6)$$

We emphasize that all quantities shown in figure 1 refer to conditions at the 'edge' of the ionization layer, where the plasma is in ionization equilibrium, and the partial composition of the plasma was evaluated by means of the Saha equation in terms of $T_{\rm e}$, $T_{\rm h}$, and the plasma pressure. The computed ionization degree of the plasma, $\omega$, approaches unity for $T_{\rm e}$ close to 2 eV, as usual for atmospheric-pressure thermal plasmas of argon (e.g. figure 1 of [21]). Remind that $\lambda_{ia}$ is defined by equation (1) and has the meaning of the mean free path for collisions between the neutral atoms and the ions; in

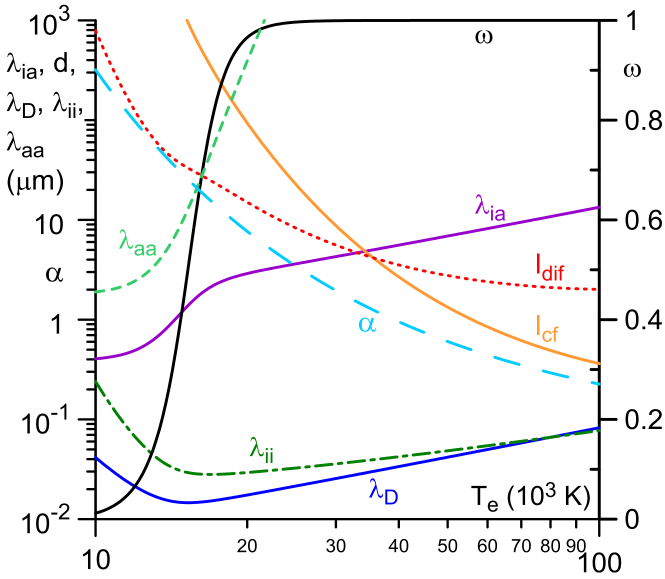


**Figure 1.** Characteristic length scales in the ionization layer in atmospheric-pressure argon arc.

the case of plasma close to full ionization it represents the mean free path of an atom in the gas of ions and remains finite, in contrast to the mean free path of an ion in the gas of atoms, which infinitely increases.

As expected, $\lambda_{aa}$ is much larger than $\lambda_{ia}$ over the entire range of interest of $T_{\rm e}$: an atom collides with the ions much more frequently than with other atoms. The following hierarchy holds on the lower end of the $T_{\rm e}$ range: $\lambda_{ia} < l_{\rm dif} < l_{\rm cf}$. It follows that the relative motion of the atoms and the ions in the ionization layer is dominated by collisions. On the higher end, $l_{\rm cf} < l_{\rm dif} < \lambda_{ia}$; the atom motion is collision-free. The three lengths are comparable for $T_{\rm e}$ approximately from 25 000 K to 50 000 K; an intermediate regime.

$\lambda_{\rm D}$ is much smaller than both $l_{\rm dif}$ and $l_{\rm cf}$. It follows that the ionization layer is quasi-neutral in all the cases and the space charge is localized in a thin layer with a scale of thickness $\lambda_{\rm D}$ adjacent to the cathode (the so-called space-charge sheath). The ionization inside the sheath is negligible, hence the ion flux to the cathode surface equals the flux of ions coming to the sheath from the ionization layer.

It is seen from figure 1 that at $T_{\rm e} \approx 34\,000$ K all the three lengths $l_{\rm dif}$, $l_{\rm cf}$, and $\lambda_{ia}$ are close to each other and $\alpha$ is around unity. This is no coincidence. It can be shown that $l_{\rm dif}^2/l_{\rm cf}\lambda_{ia} = 3\sqrt{2\pi}/16 \approx 0.83$, so if two of these lengths are close to each other, then the third one is also close. Moreover, it can be shown that

$$\frac{l_{\rm dif}}{\lambda_{ia}} = \frac{3\sqrt{\pi}}{8\sqrt{\omega}}\alpha, \quad \frac{l_{\rm cf}}{\lambda_{ia}} = \frac{3\sqrt{2}}{8\omega}\alpha^2. \quad (7)$$

It is seen from figure 1 that $\omega \approx 1$ for $T_{\rm e} \approx 34 \times 10^3$ K, hence values of $\alpha$, corresponding to $l_{\rm dif} = \lambda_{ia}$ or $l_{\rm cf} = \lambda_{ia}$ are approximately 1.5 or 1.4, respectively. On the scale of variation of $\alpha$ seen in figure 1, these values may be considered as close to unity.

It follows that the conditions of occurrence of the collision-free regime, $l_{\rm cf} \ll \lambda_{ia}$, and of the collision-dominated regime, $l_{\rm dif} \gg \lambda_{ia}$, may be expressed in terms of $\alpha$ as $\alpha \ll 1$ and $\alpha \gg 1$, respectively.

Parameter $\alpha$ is low when the ionization coefficient $k_i$ is high enough, which happens when the electron temperature in the near-cathode layer, $T_{\rm e}$, is very high. $T_{\rm e}$ is governed by the power deposited into the near-cathode layer by the electrons emitted from the cathode and accelerated by the near-cathode voltage drop. Thus, $\alpha$ is low when the cathode surface is hot, so the electron emission current is high, and simultaneously the near-cathode voltage is high. For hot cathodes, e.g. refractory cathodes operated in steady-state regimes, the near-cathode voltage is low and hence $\alpha$ exceeds unity.

On the other hand, high values of the cathode surface temperature and near-cathode voltage may simultaneously occur during glow-to-arc transitions, when a hot arc spot has already formed, but a significant part of the current still flows to the cold surface outside the spot. The near-cathode voltage continues to be high at all points of the cathode surface in such cases, including in the spot, so $\alpha$ is small in the part of the near-cathode layer adjacent to the spot. Let us consider, as an example, results of numerical simulations of quasi-stationary glow-to-arc transitions [14]. The simulations [14] were performed for a DC discharge on a 1.5 mm-radius W cathode in 1 atm Ar in the range of discharge currents $I$ up to 20 A. It was found that, as $I$ has increased to 2.5 A, a narrow spot with the temperature of 3960 K is formed and the discharge voltage drops to 60 V. Using the code [22], one finds that $\alpha$ is approximately 0.2 for such conditions. It follows from the second formula (7) that the mean free path for collisions between the atoms and the ions, $\lambda_{ia}$, exceeds the scale of thickness of the ionization layer, $l_{\rm cf}$, by a factor of about 50. Since the atom-atom mean free path $\lambda_{aa}$ is still larger, the motion of the atoms in the ionization layer may be considered as collision-free with a large margin. A similar situation was observed in numerical simulations of (non-stationary) glow-to-arc transitions occurring during ignition of a 200 A atmospheric-pressure Ar arc on a cold W insert with conical tip, surrounded by a water-cooled copper holder [23], and on a cold W rod-like cathode [24].

Thus, situations with $\alpha \lesssim 1$ do occur during glow-to-arc transitions. A theory of ionization layer for the limiting case of collisionless atom motion, $\alpha \ll 1$, is developed in the next section and a formula for the ion current to the cathode surface for arbitrary $\alpha$ is derived in section 4.

## 3. Ionization layer with collision-free atoms

This section is concerned with the limiting case $\alpha \ll 1$, where the mean free path for collisions between the atoms and the ions, $\lambda_{ia}$, is much larger than the scale of thickness of the ionization layer, $l_{\rm cf}$, and hence there is no collisional coupling between the ion and atom species. The diffusion description of the ion-atom relative motion in the layer is not valid in this case. On the other hand, the mean free path for the ion-ion collisions, $\lambda_{ii}$, is much smaller than $l_{\rm cf}$, as seen in figure 1. Hence, the ion motion may be described in the fluid approximation, as in the multifluid theory [2–6]. However, the atom-atom mean free path $\lambda_{aa}$ is large, hence the fluid description of the motion of the atoms is inaccurate and a kinetic description is more appropriate.

The ionization layer is assumed to be quasi-neutral, the space-charge sheath is assumed to be infinitely thin on the ionization layer scale. The distribution of the electron number density is related to the electrostatic potential through the Boltzmann distribution, hence the electric field may be expressed as

$$\frac{d\varphi}{dx} = \frac{kT_{\rm e}}{en_e}\frac{dn_e}{dx}. \quad (8)$$

The equations of conservation of the atoms and the ions are

$$\frac{d}{dx}(n_a v_a) = -k_i n_e n_a, \quad \frac{d}{dx}(n_i v_i) = k_i n_e n_a, \quad (9)$$

where the axis $x$ is directed from the cathode surface into the plasma and $v_a$ and $v_i$ are the $x$-projections of the mean velocities of the atoms and the ions. It is seen from figure 1 that the plasma in the ionization layer is fully ionized in the limiting case of collision-free atoms. Therefore, there is no need to take into account the recombination in the ionization layer. Since the ionization layer is assumed to be quasi-neutral, $n_e$ in equations (8) and (9) can be replaced with $n_i$. Note that equation (9), jointly with the condition that the flux of atoms desorbed from the cathode surface equals the flux of ions arriving to the surface, give the condition of zero flux of nuclei,

$$n_a v_a + n_i v_i = 0, \quad (10)$$

which should have been expected.

Since the atoms move across the ionization layer without collisions, the velocity of each atom remains constant during its lifetime from its desorption from the cathode surface to ionization. Since velocities of the atoms, being of the order of $\sqrt{kT_{\rm c}/m_i}$, are much smaller than the speed of thermal motion of the electrons, the ionization probability does not depend on the velocity of the atoms. Therefore, the velocity distribution of atoms at any point inside the ionization layer is the same as the velocity distribution of emitted atoms at the cathode surface. It follows that the average velocity of the atoms at each point of the ionization layer remains equal to the average normal velocity with which the atoms are desorbed from the cathode, $v_a = \sqrt{2kT_{\rm c}/\pi m_a}$.

The ions are treated in the fluid approximation, so $v_i$ has the meaning of the $x$-projection of the velocity of ion fluid. It is governed by the momentum conservation equation,

$$\frac{d}{dx}\left(n_i m_a v_i^2\right) = -\frac{d}{dx}(n_i k T_{\rm h}) - en_i\frac{d\varphi}{dx} + k_i n_i n_a m_a v_a. \quad (11)$$

The term on the lhs accounts for ion inertia, the terms on the rhs account for, respectively, ion pressure gradient, electric field force, and momentum transfer from the neutral atoms to the ion fluid due to ionization (but not due to elastic collisions, which are very rare); see [2–6] for further details. The electric field force term in this equation may be expressed with the use of equation (8) and combined with the ion pressure gradient term to give $-k(T_\mathrm{h}+T_\mathrm{e})\,\mathrm{d}n_i/\mathrm{d}x$.

The boundary condition for small values of $x$, i.e. in the vicinity of the space-charge sheath, is the Bohm criterion (e.g. [25, 26]): $v_i=-v_s$, where

$$v_s=\left[k(T_\mathrm{h}+T_\mathrm{e})/m_a\right]^{1/2} \tag{12}$$

is the Bohm speed. Note that the derivation of the Bohm criterion is based on the assumption that ions traverse the space-charge sheath without collisions; there is no nonarbitrary way to postulate any form of the Bohm criterion, whether fluid or kinetic, with account of collisions in the sheath ([27] and references therein). It is seen from figure 1 that $\lambda_{ia}\gg\lambda_\mathrm{D}$ under the conditions considered, hence the ion-neutral collisions are rare in the sheath. In contrast, $\lambda_{ii}$ is not much larger than $\lambda_\mathrm{D}$, especially for high $T_\mathrm{e}$. On the other hand, due to the inequality $T_\mathrm{e}\gg T_\mathrm{h}$ the ions enter the sheath with approximately the same speed close to $\sqrt{kT_\mathrm{e}/m_a}$, hence the ion-ion collisions in the sheath are also rare and the Bohm criterion is justified.

At large distances from the cathode, the plasma is fully ionized and $n_i=n_i^{(0)}=p/k(T_\mathrm{h}+T_\mathrm{e})$, where $p$ is the plasma pressure. Note that it follows from equation (10) that, since $n_a$ tends to zero at large distances from the cathode, so does $v_i$.

Transforming the last term on the rhs of equation (11) with the use of the second equation (9), one obtains

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(n_i m_a v_i^2\right)=-k(T_\mathrm{h}+T_\mathrm{e})\frac{\mathrm{d}n_i}{\mathrm{d}x}+m_a v_a\frac{\mathrm{d}}{\mathrm{d}x}(n_i v_i). \tag{13}$$

Integrating and taking into account that $n_i=n_i^{(0)}$ and $v_i=0$ at large distances from the cathode, one obtains

$$n_i m_a v_i^2=k(T_\mathrm{h}+T_\mathrm{e})\left(n_i^{(0)}-n_i\right)+m_a v_a n_i v_i. \tag{14}$$

Solving for $n_i$, one finds

$$n_i=n_i^{(0)}\frac{v_s^2}{v_s^2+v_i^2-v_i v_a}. \tag{15}$$

Remind that the ion flux to the cathode surface equals the flux of ions coming to the near-cathode space-charge sheath from the ionization layer. The latter may be expressed as $J_i=n_i|_{x=0}v_s$. Using equation (15), one finds the ion flux to the cathode in the limiting case of collision-free atoms, $\alpha\ll1$:

$$J_i=C_1 n_i^{(0)}v_s, \tag{16}$$

$$C_1=\left(2-\frac{v_a}{v_s}\right)^{-1}=\left[2-\sqrt{\frac{2}{\pi(1+\beta)}}\right]^{-1}, \tag{17}$$

where $\beta=T_\mathrm{e}/T_\mathrm{h}$.

Since the above partial integration of equations (9) and (11) has allowed the evaluation of the ion flux to the cathode, it is in principle sufficient for the purposes of this work. However, it is of interest to consider briefly the full solution. Eliminating from the second equation (9) $n_a$ by means of equation (10) and introducing dimensionless variables

$$\xi=\frac{x}{l_\mathrm{cf}},\quad N=\frac{n_i}{n_i^{(0)}},\quad V=-\frac{v_i}{v_s}, \tag{18}$$

one can rewrite the second equation (9) and expression (15) as

$$(NV)'=-N^2V, \tag{19}$$

$$N=\left[1+V^2+V\sqrt{\frac{2}{\pi(1+\beta)}}\right]^{-1}. \tag{20}$$

Here prime designates differentiation with respect to $\xi$.

Eliminating from equation (19) $N$ by means of equation (20), one obtains an equation for $V(\xi)$:

$$V'=\frac{V}{V^2-1}. \tag{21}$$

Integrating this equation with the boundary condition $V(0)=1$, one obtains

$$\xi=-\frac{1}{2}+\frac{1}{2}V^2-\ln V. \tag{22}$$

This formula jointly with expression (20) describes the distributions of the ion speed and the charged particle density in the ionization layer with collision-free atoms. An example is shown in figure 2. $V$ does not depend on $\beta$, the dependence of $N$ on $\beta$ is weak. At large values of $\xi$, i.e. at the 'edge' of the ionization layer, the plasma is fully ionized and the ion speed tends to zero proportionally to $e^{-\xi}$. Inside the layer the ions are accelerated up to the Bohm speed. The charged particle density inside the ionization layer decreases by a factor of between 2 and approximately 2.5.

Note that the asymptotic behavior for small $\xi$ of the ion speed defined by equation (22) is

$$V\approx1-\sqrt{\xi}. \tag{23}$$

It follows that for small $\xi$, i.e. in the vicinity of the space-charge sheath, the electric field, given in the quasi-neutral approximation by equation (8), increases proportionally to $1/\sqrt{\xi}$ and thus has a singularity at $\xi=0$, as usual for problems involving the Bohm criterion.

## 4. Ion current to the cathode surface for a wide range of conditions

Let us now switch from the limiting case $\alpha\ll1$, considered in the previous section, to the general case of arbitrary $\alpha$ and a plasma which is not necessarily fully ionized. Given the structure of equation (16), it is natural to represent the ion current

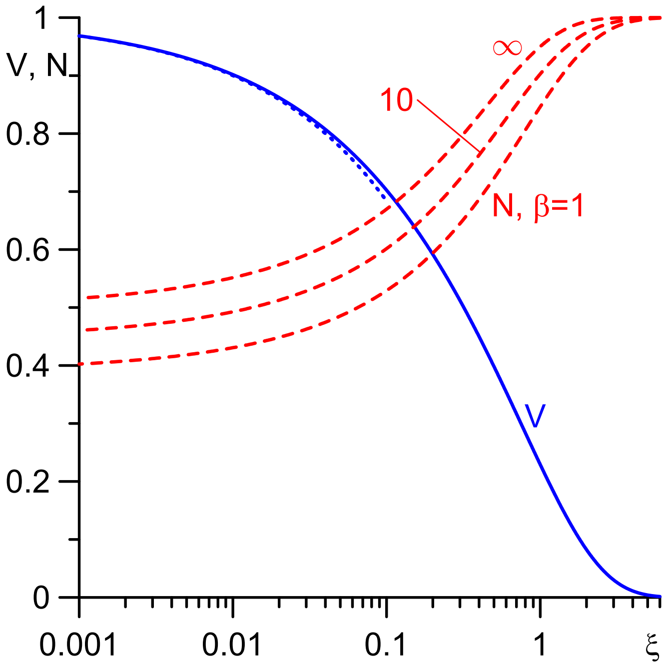


**Figure 2.** Normalized distributions of the ion speed (solid) and charged particle density (dashed) in ionization layer with collision-free atoms. Dotted: asymptotic behavior of the ion speed in the vicinity of the space-charge sheath, equation (23).

to the cathode, or, more precisely, the density of the ion flux to the cathode, as

$$J_i = n_i^{(0)} v_s f_w, \tag{24}$$

where $f_w$ is the normalized current, a quantity to be determined which depends, in particular, on $\alpha$. Remind that $n_i^{(0)}$ is the charged particle density at the 'edge' of the ionization layer, which may be readily evaluated from the Saha equation in terms of the plasma pressure, $T_h$, and $T_e$; and $v_s$ is the Bohm speed given by equation (12).

Comparing expression (24) with (16), one finds the asymptotic behavior of the dependence of $f_w$ on $\alpha$ for $\alpha \to 0$:

$$f_w \to C_1. \tag{25}$$

Note that the asymptotic expression for $f_w$ in the limiting case $\alpha \to 0$, obtained in the multifluid treatment [3], is substantially different: $f_w \approx \alpha\sqrt{1+\beta} - 2\alpha^2(1+\beta)$.

The asymptotic behavior for $\alpha \to \infty$ may be found by considering the opposite limiting case $\alpha \gg 1$, where $\lambda_{ia} \ll l_{\text{dif}}$ and the relative motion of the ions and the atoms is dominated by collisions. Under the assumption of constant $T_e$ and $T_h$, employed in this work, an analytical solution of the ambipolar diffusion equation in the ionization layer may be obtained; e.g. section II of [3]. The density of the ion flux to the cathode surface may be expressed as

$$J_i = n_i^{(0)} v_s C_2/\alpha, \tag{26}$$

where the dimensionless coefficient $C_2$ is given by the formula

$$C_2 = \left[\frac{(1+\beta)(2+\beta)}{\beta^2} - \frac{2(1+\beta)^2}{\beta^3}\ln(1+\beta)\right]^{1/2}. \tag{27}$$

In fact, equation (26) is valid for a plasma of arbitrary ionization degree provided that the expression (27) for $C_2$ is replaced by equation (14) of [28], which involves both $\beta$ and the ionization degree of the plasma in the ionization layer.

Comparing equation (26) with (24), one finds the asymptotic behavior of the dependence of $f_w$ on $\alpha$ for $\alpha \to \infty$:

$$f_w \approx C_2/\alpha. \tag{28}$$

One can write a rational interpolation for $f_w$ between the asymptotic expressions (25) and (28) (a two-point Padé approximant):

$$f_w = \frac{C_1 C_2}{C_2 + C_1\alpha}. \tag{29}$$

This formula jointly with (24) and equation (14) of [28] represents the desired approximate expression for the ion flux to the cathode surface, applicable for any $\alpha$ and partially to fully ionized plasmas.

The normalized ion current given by equation (29) is shown in figure 3 for three values of $\beta$. For definiteness, the figure refers to the case of fully ionized plasma and employs equation (27). For $\beta = 6$, also shown are the diffusion approximation, equation (28), the approximation of collision-free atoms, equation (25), and the expression for $f_w$ derived by means of the multifluid theory, equation (50) of [3]. Also shown are experimental data taken from figure 5 of [29], which were transformed as described in [4] and refer to conditions with $\beta \approx 6$.

As expected, values of the normalized ion current given by the theory of this work, by the multifluid theory, and the diffusion theory are all close to each other for large $\alpha$. For $\alpha$ of order unity and smaller, the diffusion values substantially exceed values given by the theory of this work. The multifluid theory gives lower values than the theory of this work and the difference increases as $\alpha$ decreases; a consequence of the difference in asymptotic expressions for small $\alpha$, pointed out above.

Given that the theory of this work is better justified than both the diffusion and multifluid theories, one would expect that equation (29) conforms to experiment better than both the diffusion and multifluid theories. This is clearly the case as far as the diffusion theory is concerned. This appears to be the case also for the multifluid theory, although the scatter of experimental data is too great to make an unambiguous conclusion.

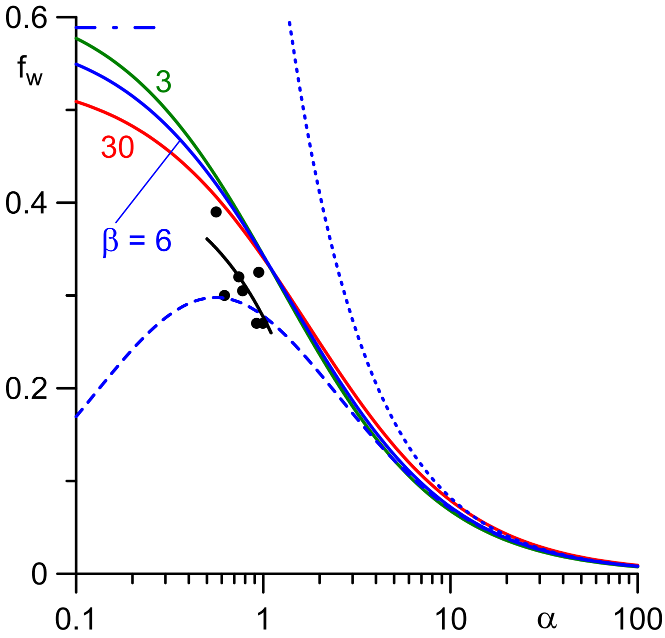


**Figure 3.** Normalized ion current to the cathode. Solid: this work, equation (29), $\beta = 3, 6, 30$. Dotted: diffusion approximation, equation (28), $\beta = 6$. Dash-dotted: approximation of collision-free atoms, equation (25), $\beta = 6$. Dashed: multifluid theory [3], $\beta = 6$. Circles and short solid line: experimental data from [29] and their linear fit.

## 5. Summary and concluding remarks

When a hot arc spot has just formed on the cathode surface, e.g. in the course of arc ignition on a cold cathode, a significant part of the current still flows in the glow-discharge mode to the cold surface outside the spot. The near-cathode voltage continues to be high at all points of the cathode surface, including in the spot. The mean free path for collisions between the atoms and the ions within the plasma ball near the spot is comparable to, or exceeds, the thickness of the ionization layer, which is a part of the near-cathode layer where the ion current to the cathode is generated. The evaluation of the ion current to the cathode surface under such conditions is revisited. A fluid description of the ion species in the ionization layer is combined with a kinetic description of the atom species. The resulting problem admits a simple analytical solution that allows one to derive formulas for evaluation of the ion current to the cathode surface for a wide range of conditions.

The results obtained can be used to improve existing methods for modeling high-pressure arc discharges and their interaction with electrodes in order to increase their accuracy in relation to glow-to-arc transitions on cold cathodes. Using the classification proposed in [7], one can distinguish four self-consistent approaches: (1) unified approach, which involves solving diffusion equations for each plasma species in the entire interelectrode gap up to the electrodes; (2) approach combining, on the one hand, diffusion equations for the arc bulk, which is assumed to be quasi-neutral, and on the other hand, models for near-cathode and near-anode space-charge sheaths; (3) approach combining, on the one hand, equations for the arc bulk, which is assumed to be quasi-neutral and in the state of ionization equilibrium but with unequal heavy-particle and electron temperatures, and on the other hand, models for near-electrode layers, which comprise the space-charge sheath and the ionization layer; and (4) approach combining, on the one hand, the LTE equations for the arc bulk, and on the other hand, models for near-electrode layers, which comprise the space-charge sheath, the ionization layer, and the layer of thermal non-equilibrium near the anode (the layer of thermal non-equilibrium near the cathode is not of primary importance).

Initially applications of the first (unified) modelling approach were limited to 1D simulations, but by now the unified approach has been extended to axially symmetric low-current arcs [12, 14, 15, 30–32]. Unfortunately, this powerful approach is unjustified for the modelling of glow-to-arc transitions, when the ionization layer is not adequately described by the diffusion approximation. An example is seen in figure 3: the diffusion approximation, equation (28), substantially overestimates the ion current to the cathode. Results of this work may be useful for analysis and eventual correction of data on glow-to-arc transitions given by the unified approach; the simplest thing would be to limit the computed ion current to the cathode by the value given by equation (29).

There are quite a few works using the second modelling approach; see citations in section 3.2 of [7] and [8, 9, 11, 13] as further examples. The near-cathode ionization layer is computed, as a part of the quasi-neutral arc bulk, in the diffusion approximation in this approach. Therefore, what was said above regarding the first approach remains valid.

The third and fourth modelling approaches require the use of a model for near-cathode non-equilibrium plasma layers in high-pressure arc discharges, comprising the ionization layer and the space-charge sheath. Such models may be used also for standalone modelling of the arc-cathode interaction. Slightly differing versions of such models are described in [10, 33–39]. Implementation of equation (29) in such models is straightforward and will be considered in a forthcoming work. The first results on the modelling of the ignition of 200 A arcs on cold tungsten cathodes of different configurations were reported in [23, 24]. More detail on the glow-to-arc transition, including the possibility of experimental verification, will be reported in a forthcoming work.

## Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

## Acknowledgments

The work was supported by FCT—Fundação para a Ciência e a Tecnologia of Portugal under Projects UIDB/50010/2020 (https://doi.org/10.54499/UIDB/50010/2020), UIDP/50010/2020 (https://doi.org/10.54499/UIDP/50010/2020), LA/P/0061/2020 (https://doi.org/10.54499/LA/P/0061/2020), and by European Regional Development Fund through the Operational Program of the Autonomous Region of Madeira 2014–2020 under Project PlasMa-M1420-01-0145-FEDER-000016.

## ORCID iD

M S Benilov https://orcid.org/0000-0001-9059-1948

## References

[1] Benilov M S 2008 *J. Phys. D: Appl. Phys.* **41** 144001
[2] Benilov M S 1995 *J. Phys. D: Appl. Phys.* **28** 286
[3] Benilov M S and Naidis G V 1998 *Phys. Rev.* E **57** 2230
[4] Almeida N A, Benilov M S, Franklin R N and Naidis G V 2004 *J. Phys. D: Appl. Phys.* **37** 3107
[5] Scharf F H and Brinkmann R P 2006 *J. Phys. D: Appl. Phys.* **39** 2738
[6] Scharf F H, Oberrath J, Merthmann P and Brinkmann R P 2007 *Proc. 11th Int. Symp. Sci. Technol. Light Sources* (*LS:11*) (*Shanghai, May 2007*) ed M Q Liu and R Devonshire (FAST-LS) pp 261–62
[7] Benilov M S 2020 *J. Phys. D: Appl. Phys.* **53** 013002
[8] Siewert E, Baeva M and Uhrlandt D 2019 *J. Phys. D: Appl. Phys.* **52** 324006
[9] Wang H-X, Zhu T, Sun S-R, Liu G and Murphy A B 2020 *J. Phys. D: Appl. Phys.* **53** 505205
[10] Mohsni C, Baeva M, Franke S, Gortschakow S, Araoud Z and Charrada K 2020 *Phys. Plasmas* **27** 073514
[11] Zhang H, Zhang H, Trenchev G, Li X, Wu Y and Bogaerts A 2020 *Plasma Sources Sci. Technol.* **29** 045019
[12] Baeva M, Uhrlandt D and Loffhagen D 2020 *Jpn. J. Appl. Phys.* **59** SHHC05
[13] Sun J-H, Sun S-R, Niu C and Wang H-X 2021 *J. Phys. D: Appl. Phys.* **54** 465202
[14] Saifutdinov A I 2022 *Plasma Sources Sci. Technol.* **31** 094008
[15] Tsonev I, Boothroyd J, Kolev S and Bogaerts A 2023 *Plasma Sources Sci. Technol.* **32** 054002
[16] Almeida R M S, Benilov M S and Naidis G V 2000 *J. Phys. D: Appl. Phys.* **33** 960
[17] Benilov M S, Almeida N A, Baeva M, Cunha M D, Benilova L G and Uhrlandt D 2016 *J. Phys. D: Appl. Phys.* **49** 215201
[18] Chapman S and Cowling T G 1970 *The Mathematical Theory of Non-Uniform Gases* 3rd edn (Cambridge University Press)
[19] Hirschfelder J O, Curtiss C F and Bird R B 1964 *Molecular Theory of Gases and Liquids* (Wiley)
[20] Ferziger J H and Kaper H G 1972 *Mathematical Theory of Transport Processes in Gases* (North-Holland)
[21] Colombo V Ghedini E and Sanibondi P 2008 *Prog. Nucl. Energy* **50** 921
[22] Benilov M S and Cunha M D 2019 NCPL - On-line tool for evaluation of parameters of non-equilibrium near-cathode plasma layer in high-pressure arc plasmas (available at: https://fisica.uma.pt/public-domain/simulation-tools/ncpl/)
[23] Cunha M D, Sargsyan M A, Gadzhiev M K, Tereshonok D V and Benilov M S 2023 *J. Phys. D: Appl. Phys.* **56** 395204
[24] Ojeda O, Cressault Y, Teulet P, Gonnet J-P, Santos D F N, Cunha M D and Benilov M S 2023 *23rd Int. Conf. Gas Discharges and Their Applications* vol 1 pp 64–67
[25] Franklin R N 1976 *Plasma Phenomena in Gas Discharges* (Clarendon)
[26] Lieberman M A and Lichtenberg A J 2005 *Principles of Plasma Discharges and Material Processing* 2nd edn (Wiley)
[27] Benilov M S and Almeida N A 2019 *Phys. Plasmas* **26** 123505
[28] Benilov M S 1999 *J. Phys. D: Appl. Phys.* **32** 257
[29] Baksht F G, Dyuzhev G A, Mitrofanov N K, Shkol'nik S M and Yur'ev V G 1973 *Sov. Phys. Tech. Phys.* **18** 1617
[30] Kolev S and Bogaerts A 2015 *Plasma Sources Sci. Technol.* **24** 015025
[31] Saifutdinov A I, Fairushin I I and Kashapov N F 2016 *JETP Lett.* **104** 180
[32] Kolev S, Sun S, Trenchev G, Wang W, Wang H and Bogaerts A 2017 *Plasma Process. Polym.* **14** 1600110
[33] Benilov M S, Cunha M D and Naidis G V 2005 *Plasma Sources Sci. Technol.* **14** 517
[34] Wendelstorf J 1999 *Contributed Papers of 24th Int. Conf. on Phenomena in Ionized Gases* (*Warsaw 1999*) vol 2, ed P Pisarczyk, T Pisarczyk and J Wolowski (Institute of Plasma Physics and Laser Microfusion) pp 227–8 (Available from the British Library)
[35] Schmitz H and Riemann K-U 2002 *J. Phys. D: Appl. Phys.* **35** 1727
[36] Lichtenberg S, Dabringhausen L, Langenscheidt O and Mentel J 2005 *J. Phys. D: Appl. Phys.* **38** 3112
[37] Scharf F H, Langenscheidt O and Mentel J 2007 *Proc. 28th ICPIG* (*Prague, July 2007*) ed J Schmidt, M Šimek, S Pekárek and V Prukner (Institute of Plasma Physics AS CR) pp 1252–5
[38] Sun Q, Wang C, Chen T and Xia W-D 2017 *J. Phys. D: Appl. Phys.* **50** 425202
[39] Mohsni C, Baeva M, Franke S, Gortschakow S, Gonzalez D, Araoud Z and Charrada K 2019 *Plasma Phys. Technol.* **6** 51