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Selective Base Revisions

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Abstract Belief Revision addresses the problem of rationally incorporating pieces of new information into an agent's belief state. In the AGM paradigm, the most used framework in Belief Revision, primacy is given to the new information, which is fully incorporated into the agent's belief state. However, in real situations, one may want to reject the new information or only accept a part of it. A constructive model called Selective Revision was proposed to meet this need but, as in the AGM framework, focused on belief sets (sets closed under logical consequence). In this paper we adapt the selective revision operators, that were proposed for belief sets, to the belief base context, obtaining a model in which an agent's epistemic state is represented by a belief base and that allows the acceptance of only part of the new information. We present several representation theorems for selective base revision operators based on different base revision operators.

Keywords Belief Revision · Non-prioritized Belief Revision · Selective Revisions · Belief Bases

1 Introduction

One of the main goals underlying the research area of *belief change* consists in finding appropriate ways of modelling how the belief state of an agent is modified when he/she receives some new information. One of the main contributions to the study of belief change is the so-called AGM model for belief change ([1]). In that framework, each *belief* of an agent is represented by a sentence and the *belief state* of an agent is represented by a logically

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closed set of (belief-representing) sentences, called *belief sets*. The AGM model considers three kinds of belief change operators, namely *expansion, contraction* and *revision*. An expansion occurs when new information is simply added to the set of the beliefs of an agent. A contraction consists in removing beliefs from the agent's set of beliefs until the input information is no longer derived from it. A revision occurs when the new belief is added in a consistent way (whenever that is possible), possibly demanding a repair in order to eliminate inconsistency.

Although the AGM model has acquired the status of a standard model of belief change, several researchers (for an overview see [7] and [8]) have pointed out its inadequateness in several contexts and proposed several extensions and generalizations to that framework.

One criticism pointed out to the AGM framework concerns the use of logically closed sets (or belief sets) to model the belief state of an agent. Belief sets are very large entities, and its use is not adequate for computational implementations. The logical closure of belief sets raises other issues not related to computational implementations. If we intend to model the way humans reason the AGM model is inadequate. Rott pointed out in [29] that the AGM theory is unrealistic in its assumption that epistemic agents are "ideally competent regarding matters of logic. They should accept all the consequences of the beliefs they hold (that is, their set of beliefs should be logically closed), and they should rigorously see to it that their beliefs are consistent". In the AGM framework agents have unlimited memory and ability of inference, this is an unrealistic feature of the human reasoning.

A set A is a *base* for a belief set $\mathbf{K}$ if and only if $Cn(A) = \mathbf{K}$. The modelled agent explicitly believes a sentence if and only if that sentence is in A; the agent is committed to the truth of a sentence if and only if that sentence is in $Cn(A)$. Belief bases provide more structure to the epistemic state of an agent: they allow the distinction between two different inconsistent belief states as well as the distinction between beliefs that have independent standing (beliefs in the belief base A) from those that are merely derived beliefs (beliefs in $Cn(A) \setminus A$). Merely derived beliefs are automatically removed when the beliefs of the underlying belief base that support them are withdrawn. The use of belief bases has been largely studied in the literature [4,11,16,17,19,22,31]. *Partial meet base contractions*, were presented in [18,20,17]; *kernel contractions* and *smooth kernel contractions* were introduced in [21]; and *basic AGM-generated base contractions* were studied in [9]. *Partial meet base revisions*, were studied in [17]; *kernel* and *smooth kernel revisions* were introduced in [21] and axiomatically characterized in [31] and [12], respectively; and *basic AGM-generated base revisions* were proposed in [12].

Another feature of the AGM model is the unconditional acceptance of the incoming information. In this model the new information outweighs the current one and hence, in case of conflict, the former will prevail. This behaviour might be a reasonable assumption in many situations. If a new observation contradicts a scientific theory, the theory should be adapted to account for it; if a law changes then the new decisions should be ruled by the newest version of the law. However, there are cases in which this feature, characterized by the *success* postulate, is unrealistic. As stated in [10], "Rational agents, when confronted with information that contradicts previous beliefs, often reject it altogether or accept only parts of it". This may happen for various reasons. For example, the new information may lack on credibility or it may contradict previous highly entrenched beliefs. Belief revision operators that do not satisfy the *success postulate* are called *non-prioritized belief revisions*.

There are in the literature several models of *non-prioritized belief revisions*. Credibility-limited revision ([26]), is an operator which, roughly speaking, has the following behaviour (that relies on the notion of *credibility of a sentence*): If a sentence α is credible, then a credibility-limited revision by it works as a standard AGM revision, otherwise no change is made to the belief set. In [10] and [12] the model of credibility-limited revision was adapted to the belief base context. In Screened revision ([27]) instead of a set of credible sentences it is considered a set A of sentences that are immune to revision and a revision of a belief set $\mathbf{K}$ by a given sentence α only gives rise to a new (appropriately changed) belief set if the input sentence α is consistent with $A \cap \mathbf{K}$ (otherwise the belief set $\mathbf{K}$ is left unchanged).

In [6], Fermé and Hansson proposed an operator, for belief sets, that allows the acceptance of only part of the new information and the rejection of the rest of it. They called this operator *selective revision*. The following example, motivates the proposal of this operator:

Example 1 ([6]) Suppose one day when I get home my youngest daughter tells me that a dinosaur broke her grandmother's vase in the living room. I will probably accept part of the information, namely that the vase is broken and I will certainly reject the part concerning the dinosaur.[1]

An operator of selective revision, $\circledast$, is constructed from a basic AGM revision $\star$ and a transformation function f from $\mathcal{L}$ to $\mathcal{L}$ as follows:

$$\mathbf{K} \circledast \alpha = \mathbf{K} \star f(\alpha).$$

Intuitively, the transformation function f selects the credible part of every sentence. When adding properties to the transformation function f, several additional properties are obtained for the selective revision operator which is based on f. In [6] several representation theorems for operators of selective revision were presented depending on the properties that the associated transformation function f satisfies.

In the present paper we shall study selective revision operators defined for belief bases (rather than for belief sets). This paper significantly extends part of the study carried out in [28]. In this article we consider classes of selective revision operators based on different kinds of base revisions, namely partial meet revisions, (smooth) kernel revisions and basic AGM-generated base revisions. We present several representation theorems for those classes

[1] In this example, the new information is considered to be "a dinosaur broke my youngest daughter's grandmother's vase in the living room" and not "my youngest daughter just told me that a dinosaur broke her grandmother's vase in the living room" (if that were the case, then the new information should be fully incorporated).

of operators. With this article, we intend to provide further mathematical foundations of selective revision.[2]

The rest of the paper is organized as follows: In Section 2 we introduce the notations and recall the main background concepts that will be needed throughout this article. In Section 3 we present a formal definition of selective base revision and introduce some desirable properties that transformation functions should satisfy. We present several results highlighting the interrelations among postulates of selective base revision and the properties satisfied by the (standard) revision and transformation functions on which the former is based. At the end of Section 3, we present representation theorems for selective base revision based on different types of base revision operators, namely partial meet revisions, (smooth) kernel revisions and basic AGM-generated base revisions. In Section 4 we summarize the main contributions of the paper, briefly discuss their relevance and present some potential future research topics. In the Appendix we provide proofs for all the original results presented.

2 Background

2.1 Formal preliminaries

We will assume a language $\mathcal{L}$ that contains the usual truth functional connectives: $\neg$ (negation), $\wedge$ (conjunction), $\vee$ (disjunction), $\rightarrow$ (implication) and $\leftrightarrow$ (equivalence). A Tarskian consequence operator Cn will be used. Intuitively speaking, $Cn(A)$ is the set of logical consequences of A. More formally, a consequence operation Cn is a function that takes sets of sentences to sets of sentences and which satisfies the standard Tarskian properties ([30]), namely: (i) $A \subseteq Cn(A)$ (*inclusion*); (ii) If $A \subseteq B$, then $Cn(A) \subseteq Cn(B)$ (*monotony*) and (iii) $Cn(A) = Cn(Cn(A))$ (*iteration*). Furthermore we will assume that Cn satisfies the following three properties: (iv) If α can be derived from A by classical truth-functional logic, then $\alpha \in Cn(A)$ (*supraclassicality*); (v) $\beta \in Cn(A \cup \{\alpha\})$ if and only if $\alpha \rightarrow \beta \in Cn(A)$ (*deduction*) and (vi) If $\alpha \in Cn(A)$, then $\alpha \in Cn(A')$ for some finite subset A' of A (*compactness*). We will sometimes use $Cn(\alpha)$ for $Cn(\{\alpha\})$, $A \vdash \alpha$ for $\alpha \in Cn(A)$, $\vdash \alpha$ for $\alpha \in Cn(\emptyset)$, $A \nvdash \alpha$ for $\alpha \notin Cn(A)$, $\nvdash \alpha$ for $\alpha \notin Cn(\emptyset)$, $\alpha \vdash \beta$ for $\{\alpha\} \vdash \beta$. The letters $\alpha, \beta, \ldots$ will be used to denote sentences of $\mathcal{L}$. Uppercase Latin letters such as $A, B, \ldots$ shall denote sets of sentences of $\mathcal{L}$. $\mathbf{K}$ is reserved to represent a set of sentences that is closed under logical consequence (*i.e.* $\mathbf{K} = Cn(\mathbf{K})$) — such a set is called a *belief set* or *theory*. Given a set A we will denote $Cn(A \cup \{\alpha\})$ by $A + \alpha$.

[2] It may also serve to complement the philosophical foundations of this kind of operation that might come in the future.

2.2 AGM revisions

The operation of revision of a belief set consists of incorporating new beliefs into that set. In a revision process, some previous beliefs may be retracted in order to obtain (if possible), as output, a consistent belief set. The following six postulates, which were originally presented in [13], are commonly known as *basic AGM postulates for revision*:[3]

($\star$1) $\mathbf{K}\star\alpha = Cn(\mathbf{K}\star\alpha)$ (*i.e.* $\mathbf{K}\star\alpha$ is a belief set). (Closure)
($\star$2) $\alpha \in \mathbf{K}\star\alpha$. (Success)
($\star$3) $\mathbf{K}\star\alpha \subseteq \mathbf{K} + \alpha$. (Inclusion)
($\star$4) If $\neg\alpha \notin \mathbf{K}$, then $\mathbf{K} + \alpha \subseteq \mathbf{K}\star\alpha$. (Vacuity)
($\star$5) If α is consistent, then $\mathbf{K}\star\alpha$ is consistent. (Consistency)
($\star$6) If $\vdash \alpha \leftrightarrow \beta$, then $\mathbf{K}\star\alpha = \mathbf{K}\star\beta$. (Extensionality)

Definition 1 An operator $\star$ for a belief set $\mathbf{K}$ is:

- a basic AGM revision if and only if it satisfies postulates ($\star$1) to ($\star$6);
- an AGM revision if and only if it satisfies postulates ($\star$1) to ($\star$6),
  (Disjunctive Overlap) [13] $\mathbf{K}\star\alpha \cap \mathbf{K}\star\beta \subseteq \mathbf{K}\star(\alpha \vee \beta)$,
  (Disjunctive Inclusion) [13] If $\neg\alpha \notin \mathbf{K}\star(\alpha\vee\beta)$, then $\mathbf{K}\star(\alpha\vee\beta) \subseteq \mathbf{K}\star\alpha$.

2.3 Selective revisions on belief sets

In a standard (basic) AGM revision the new information is always fully accepted and incorporated in the agent's set of beliefs. Fermé and Hansson ([6]) proposed an operator that allows the acceptance of only part of the new information and the rejection of the rest of it. They called this operator *selective revision*. In this subsection we recall from [6] the definition of this kind of operator and representation theorems for two different classes of such operators. An operator of selective revision is constructed from a basic AGM revision and a function from $\mathcal{L}$ to $\mathcal{L}$:

Definition 2 ([6]) Let $\mathbf{K}$ be a belief set, $\star$ be a basic AGM revision operator for $\mathbf{K}$ and f be a function from $\mathcal{L}$ to $\mathcal{L}$. The selective revision $\odot$, based on $\star$ and f, is the operation such that for all sentences α:

$$\mathbf{K} \odot \alpha = \mathbf{K}\star f(\alpha).$$

f is the transformation function on which $\odot$ is based.

Intuitively, the transformation function f selects the credible part of every sentence. A natural restriction is that $f(\alpha)$ should only contain information that is present in α, *i.e.* it should hold that $\vdash \alpha \to f(\alpha)$. The following are

[3] The postulates were already presented in [1] but with slightly different formulations.

some of the properties for transformation functions presented in [6]:[4]

$\vdash \alpha \rightarrow f(\alpha)$	(Implication)
$\vdash f(f(\alpha)) \leftrightarrow f(\alpha)$	(Idempotence)
If $\vdash \alpha \leftrightarrow \beta$, then $\vdash f(\alpha) \leftrightarrow f(\beta)$	(Extensionality)
If $\nvdash \neg\alpha$, then $\nvdash \neg f(\alpha)$	(Consistency Preservation)
$\nvdash \neg f(\alpha)$	(Consistency)
If $\mathbf{K} \nvdash \neg\alpha$, then $\vdash f(\alpha) \leftrightarrow \alpha$	(Weak Maximality)

We now recall the following postulates that were proposed in [6] for selective revisions.

(Weak success) If $\mathbf{K} \nvdash \neg\alpha$, then $\mathbf{K} \odot \alpha \vdash \alpha$.
(Proxy success) There is a sentence β, such that $\mathbf{K} \odot \alpha \vdash \beta$, $\vdash \alpha \rightarrow \beta$ and $\mathbf{K} \odot \alpha = \mathbf{K} \odot \beta$.
(Weak proxy success) There is a sentence β, such that $\mathbf{K} \odot \alpha \vdash \beta$ and $\mathbf{K} \odot \alpha = \mathbf{K} \odot \beta$.
(Consistent expansion) If $\mathbf{K} \nsubseteq \mathbf{K} \odot \alpha$, then $\mathbf{K} \cup (\mathbf{K} \odot \alpha) \vdash \perp$.

Weak success, proxy success and *weak proxy success* are weaker versions of *success. Proxy success* captures the intuition that the revision should take the form of accepting some part of the incoming information. *Consistent expansion* expresses the idea that previous beliefs are given up in order to avoid inconsistency. As stated in [6], *consistent expansion* follows from *success* and *vacuity.*

We finish this subsection by recalling from [6] representation theorems for two classes of selective revision functions.

Theorem 1 ([6]) *Let* $\mathbf{K}$ *be a belief set and* $\odot$ *be an operator on* $\mathbf{K}$*. Then the following conditions are equivalent:*[5]

(a) $\odot$ *satisfies closure, inclusion, vacuity, consistency, extensionality and weak proxy success.*
(b) There exists a basic AGM revision operator $\star$ *for* $\mathbf{K}$ *and a transformation function* f *that satisfies extensionality, consistency preservation, weak maximality and idempotence such that* $\mathbf{K} \odot \alpha = \mathbf{K} \star f(\alpha)$*, for all* α*.*

Theorem 2 ([6]) *Let* $\mathbf{K}$ *be a belief set and* $\odot$ *be an operator on* $\mathbf{K}$*. Then the following conditions are equivalent:*

[4] We note that the designations given to some of the properties of transformation functions are the same as those assigned to postulates. Along this article, whenever these designations are used it will be clear from the context whether these refer to properties of transformations functions or to postulates.

[5] In [6], *consistent expansion* is also included in the list of postulates stated in (a), but this postulates is redundant since it follows from *weak proxy success* and *vacuity.* To see this, assume that $\mathbf{K} \nsubseteq \mathbf{K} \odot \alpha$. By *weak proxy success* there exists a sentence β, such that $\mathbf{K} \odot \alpha \vdash \beta$ and $\mathbf{K} \odot \alpha = \mathbf{K} \odot \beta$. It follows by *vacuity* that $\neg\beta \in \mathbf{K}$ (otherwise it would hold that $\mathbf{K} \subseteq \mathbf{K} \odot \beta = \mathbf{K} \odot \alpha$). Thus $\mathbf{K} \cup (\mathbf{K} \odot \alpha) \vdash \perp$.

(a) $\odot$ *satisfies closure, inclusion, vacuity, consistency, extensionality and proxy success.*
(b) *There exists a basic AGM revision operator* $\star$ *for* $\mathbf{K}$ *and a transformation function* f *that satisfies extensionality, consistency preservation, weak maximality, idempotence and implication such that* $\mathbf{K} \odot \alpha = \mathbf{K} \star f(\alpha)$*, for all* α*.*

2.4 Base revisions

One of the issues encountered when trying to use AGM revisions is the need to work with logically closed (and potentially infinite) belief sets. Belief base revisions, on the other hand, deal with sets not necessarily closed. In this subsection, we recall from the literature some constructions for belief base revisions and their representation theorems.

Definition 3 An operator $*$ for a set A is an operator of revision if and only if $*$ satisfies the following postulates:
(Success) $\alpha \in A * \alpha$.
(Inclusion) $A * \alpha \subseteq A \cup \{\alpha\}$.
(Consistency) If $\alpha \nvdash \bot$, then $A * \alpha \nvdash \bot$.

2.4.1 Partial meet revisions

We now recall the definition and an axiomatic characterization of partial meet revisions. The construction of such operators is based on the concept of *remainder set*, that is the set of maximal subsets (of a given set) that fail to imply a given sentence. Formally:

Definition 4 ([2]) Let A be a belief base and α a sentence. The set $A \bot \alpha$ (A remainder α) is the set of sets such that $B \in A \bot \alpha$ if and only if:
(i) $B \subseteq A$; (ii) $B \nvdash \alpha$ and (iii) There is no set B' such that $B \subset B' \subseteq A$ and $B' \nvdash \alpha$.

The partial meet contractions are obtained by intersecting some elements of the (associated) remainder set. The choice of those elements is performed by a *selection function*. A selection function associates to each non-empty remainder set one of its non-empty subsets and to the empty (remainder) set the singleton set $\{A\}$.

Definition 5 ([1]) Let A be a belief base. A selection function for A is a function γ such that for all sentences α:
(i) If $A \bot \alpha$ is non-empty, then $\gamma(A \bot \alpha)$ is a non-empty subset of $A \bot \alpha$.
(ii) If $A \bot \alpha$ is empty, then $\gamma(A \bot \alpha) = \{A\}$.

A partial meet contraction is obtained by intersecting the elements chosen by a selection function. Partial meet revisions can be obtained by adding the sentence by which the set is revised to the set which is the outcome of the contraction of the original belief base by the negation of that sentence.

Definition 6 ([1]) Let A be a belief base. The partial meet revision operator on A based on a selection function γ is the operator $*_\gamma$ such that for all sentences α :

$$A *_\gamma \alpha = (\bigcap \gamma(A \perp \neg\alpha)) \cup \{\alpha\}.$$

An operator $*$ on A is a partial meet revision if and only if there is a selection function γ for A such that for all sentences α: $A * \alpha = A *_\gamma \alpha$.

The following observation recalls an axiomatic characterization for partial meet base revisions.

Observation 3 ([17]) *Let A be a belief base. An operator $*$ on A is a partial meet revision if and only if $*$ satisfies success, consistency, inclusion,*
(Uniformity) [20] *If for all subsets $A' \subseteq A, A' \cup \{\alpha\} \vdash \perp$ if and only if $A' \cup \{\beta\} \vdash \perp$, then $A \cap (A * \alpha) = A \cap (A * \beta)$,*
(Relevance) [20] *If $\beta \in A$ and $\beta \notin A * \alpha$, then there is some A' such that $A * \alpha \subseteq A' \subseteq A \cup \{\alpha\}, A' \nvdash \perp$ but $A' \cup \{\beta\} \vdash \perp$.*

2.4.2 Kernel revisions

In [21] Hansson introduced the *kernel contraction*, a generalization of *safe contraction* ([3]). It is based on a selection among the sentences of a set A that contribute effectively to imply α; and on how to use this selection in the process of contracting by α.

Definition 7 ([21]) Let A be a set in $\mathcal{L}$ and α be a sentence. Then $A \perp\!\!\!\perp \alpha$ is the set such that $B \in A \perp\!\!\!\perp \alpha$ if and only if: (i)$B \subseteq A$; (ii) $B \vdash \alpha$ and (iii) If $B' \subset B$ then $B' \nvdash \alpha$. $A \perp\!\!\!\perp \alpha$ is called the kernel set of A with respect to α and its elements are the α-kernels of A.

An α-kernel can be described as a minimal part of the belief base that is strong enough to commit the belief base to the truth of α; so, in order to free the belief base from the commitment to the truth of α, we need to remove at least one element from each α-kernel. An incision function is an operation that (a) chooses a (to-be-removed) non-empty subset of each non-empty α-kernel and then (b) outputs the union of the chosen subsets.

Definition 8 ([21]) Let A be a set of sentences. Let $A \perp\!\!\!\perp \alpha$ be the kernel set of A with respect to α. An incision function σ for A is a function such that for all sentences α: (i) $\sigma(A \perp\!\!\!\perp \alpha) \subseteq \bigcup(A \perp\!\!\!\perp \alpha)$; (ii) If $\emptyset \neq B \in A \perp\!\!\!\perp \alpha$, then $B \cap \sigma(A \perp\!\!\!\perp \alpha) \neq \emptyset$.

The outcome of a kernel contraction is the set of all elements of the original set that were not selected for removal by the incision function. Kernel revisions can be obtained by adding the sentence by which the set is revised to the outcome of the kernel contraction of the original set by the negation of that sentence.

Definition 9 Let A be a belief base. The kernel revision operator on A based on an incision function σ is the operator $*_\sigma$ such that for all sentences α:

$$A *_\sigma \alpha = (A \setminus \sigma(A \perp\!\!\!\perp \neg\alpha)) \cup \{\alpha\}.$$

The following observation presents an axiomatic characterization for kernel base revisions.

Observation 4 ([31]) *Let A be a belief base. An operator $*$ on A is a kernel revision function for A if and only if $*$ satisfies success, consistency,*[6] *inclusion, uniformity and*
(Core-retainment) [31] *If $\beta \in A$ and $\beta \notin A * \alpha$, then there is some $A' \subseteq A$ such that $A' \nvdash \neg\alpha$ and $A' \cup \{\beta\} \vdash \neg\alpha$.*

The following definition introduces the concept of smooth kernel base revision which is a kernel base revision based on a smooth incision function.

Definition 10 ([21]) An incision function σ for a set A is smooth if and only if it holds for all subsets A' of A that if $A' \vdash \beta$ and $\beta \in \sigma(A \perp\!\!\!\perp \alpha)$ then $A' \cap \sigma(A \perp\!\!\!\perp \alpha) \neq \emptyset$. A kernel revision is smooth if and only if it is based on a smooth incision function.

In the following observation an axiomatic characterization is provided for smooth kernel base revisions.

Observation 5 ([12]) *Let A be a belief base. An operator $*$ on A is a smooth kernel revision if and only if it satisfies success, consistency, inclusion, uniformity, core-retainment and*
(Weak relative closure) [21,12] $A \cap Cn(A \cap A * \alpha) \subseteq A * \alpha$.

2.4.3 Basic AGM-generated base revisions

We will now recall the definition and an axiomatic characterization for *basic AGM-generated base revisions*, which are operators of base revision defined from operators of basic AGM revision (for belief sets).

Definition 11 ([12]) Let A be a belief base. An operator $*$ for A is a basic AGM-generated base revision if and only if there exists some basic AGM revision $\star$ for $Cn(A)$, such that for all $\alpha \in \mathcal{L}$:

$$A * \alpha = (Cn(A) \star \alpha) \cap (A \cup \{\alpha\}).$$

[6] To be more precise we note that this axiomatic characterization is equivalent to the one actually presented in [31], which uses the postulate of *non-contradiction* (if $\nvdash \neg\alpha$, then $A * \alpha \nvdash \neg\alpha$) instead of *consistency*.

Observation 6 ([12]) *Let A be a belief base. An operator $*$ on A is a basic AGM-generated base revision if and only if it satisfies success, consistency, inclusion,*
(Vacuity) *If $A \nvdash \neg\alpha$, then $A \cup \{\alpha\} \subseteq A * \alpha$,*
(Relative extensionality) [12] *If $\vdash \alpha \leftrightarrow \beta$, then $A \cap A * \alpha = A \cap A * \beta$,*
(Disjunctive elimination) [9,12] *If $\beta \in A$ and $\beta \notin A*\alpha$, then $A*\alpha \nvdash \neg\alpha \vee \beta$.*

3 Selective revisions on belief bases

In this section we present a formal definition of selective base revision and introduce some desirable properties that a transformation function should satisfy. At the end of this section, we present representation theorems for selective base revision based on different types of base revision operators.

Definition 12 Let A be a belief base, $*$ be a base revision operator on A and f a function from $\mathcal{L}$ to $\mathcal{L}$. The selective revision based on $*$ and f is the operation $\circledast$ such that for all sentences α:

$$A \circledast \alpha = A * f(\alpha)$$

f is the transformation function on which $\circledast$ is based.

We now present a list of properties that the transformation function may be expected to satisfy:

$\vdash \alpha \rightarrow f(\alpha)$ (Implication)
$f(f(\alpha)) = f(\alpha)$ (Idempotence)
If $\nvdash \neg\alpha$, then $\nvdash \neg f(\alpha)$ (Consistency preservation)
$\nvdash \neg f(\alpha)$ (Consistency)
If $A \nvdash \neg\alpha$, then $f(\alpha) = \alpha$ (Weak Maximality)
If $A \vdash \alpha$, then $f(\alpha) = \alpha$ (Strong lower boundary)
If $\alpha \in A$, then $f(\alpha) = \alpha$ (Lower boundary)
If $A \nvdash \perp$, then $\nvdash \neg f(\alpha)$ (Inherited consistency)
If for all subsets $A' \subseteq A$, $A' \cup \{\alpha\} \vdash \perp$ if and only if $A' \cup \{\beta\} \vdash \perp$, then $f(\alpha) = \alpha$ if and only if $f(\beta) = \beta$. (Uniform identity)
If $\vdash \alpha \leftrightarrow \beta$, then $f(\alpha) = \alpha$ if and only if $f(\beta) = \beta$. (Equivalence propagation)

The first five properties were already recalled in Subsection 2.3 or result from those by adapting the namesake property to the belief base context. We note if a transformation function satisfies *implication*, then it also satisfies *consistency preservation*. If f satisfies *strong lower boundary* then a selective revision that is based on f is such that, when revising by a consequence of the agent's beliefs that sentence must be fully accepted. *Lower boundary* states that an agent's explicit belief should be in the outcome of the selective revision by it. *Inherited consistency* states that the credible part of each belief is consistent whenever the agent's set of beliefs is consistent. *Uniform identity* states that if two beliefs are inconsistent with exactly the same subsets of A, then one of them is considered to be (fully) credible if and only if the same thing happens regarding the other one. *Equivalence propagation* states that if two sentences are logically equivalent then either both are fully credible or none of them is so.

3.1 Postulates for selective revisions on belief bases

We start this subsection by presenting the postulates that we will use in the representation theorems for the selective base revisions that we will present further ahead.

The first set of postulates were already recalled in Subsection 2.4.

(Consistency) If $\alpha \nvdash \bot$, then $A \circledast \alpha \nvdash \bot$.
(Vacuity) If $A \nvdash \neg\alpha$, then $A \cup \{\alpha\} \subseteq A \circledast \alpha$.
(Weak relative closure) $A \cap Cn(A \cap A \circledast \alpha) \subseteq A \circledast \alpha$.

The next set of postulates are weaker versions of *success*.

(Proxy success) There is a sentence β, such that $\beta \in A \circledast \alpha$, $\vdash \alpha \rightarrow \beta$ and $A \circledast \alpha = A \circledast \beta$.
(Weak proxy success) There is a sentence $\beta \in A \circledast \alpha$ and $A \circledast \alpha = A \circledast \beta$
(Stability) If $\alpha \in A$, then $\alpha \in A \circledast \alpha$.
(Strong stability) If $A \vdash \alpha$, then $\alpha \in A \circledast \alpha$.
(Uniform success) If for all subsets $A' \subseteq A, A' \cup \{\alpha\} \vdash \bot$ if and only if $A' \cup \{\beta\} \vdash \bot$, then $\alpha \in A \circledast \alpha$ if and only if $\beta \in A \circledast \beta$.
(Extensional success) If $\vdash \alpha \leftrightarrow \beta$, then $\alpha \in A \circledast \alpha$ if and only if $\beta \in A \circledast \beta$.

Proxy success and *weak proxy success* were already presented in Subsection 2.3. *Stability* states that explicit beliefs of an agent (*i.e.*, that are in the agents set of beliefs) are in the outcome of a selective revision by that belief. *Strong stability* ([24]) states that all the agent's beliefs (explicit or merely derived ones) are in the outcome of a selective revision by that belief. *Uniform success* states that if two sentences are inconsistent with exactly the same subsets of A, then one of them is incorporated in the outcome of the selective revision by it if and only if the same thing happens with the other. *Extensional success*, which is a weaker version of *uniform success* states that a belief is accepted and incorporated when performing a selective revision by it if and only if the same thing happens with its logical equivalent beliefs.

The following two postulates are related to consistency. *Consistency preservation* was proposed in [27] and states that the outcome of a selective revision on a belief base A is consistent whenever A is consistent. *Strong consistency*

([23]) states that the outcome of a selective revision is consistent.

(Consistency preservation) If $A \not\vdash \bot$, then $A \circledast \alpha \not\vdash \bot$.
(Strong consistency) $A \circledast \alpha \not\vdash \bot$.

The following postulates are weaker versions of the postulates for base revision presented in Subsection 2.4. These postulates are preconditioned by $\alpha \in A \circledast \alpha$. Informally this means that if a sentence is in the outcome of a selective revision by it, then that outcome coincides with that obtained through a standard revision.

(Weak inclusion) If $\alpha \in A \circledast \alpha$, then $A \circledast \alpha \subseteq A \cup \{\alpha\}$.
(Weak vacuity) If $\alpha \in A \circledast \alpha$ and $A \not\vdash \neg\alpha$, then $A \cup \{\alpha\} \subseteq A \circledast \alpha$.
(Weak uniformity) If $\alpha \in A \circledast \alpha$ and for all subsets A' of A it holds that $A' \cup \{\alpha\} \vdash \bot$ if and only if $A' \cup \{\beta\} \vdash \bot$, then $A \cap (A \circledast \alpha) = A \cap (A \circledast \beta)$.
(Weak relevance) If $\alpha \in A \circledast \alpha$, $\beta \in A$ and $\beta \notin A \circledast \alpha$, then there is some A' such that $A \circledast \alpha \subseteq A' \subseteq A \cup \{\alpha\}$, $A' \not\vdash \bot$ but $A' \cup \{\beta\} \vdash \bot$.
(Weak core-retainment) If $\alpha \in A \circledast \alpha$, $\beta \in A$ and $\beta \notin A \circledast \alpha$, then there is some $A' \subseteq A$ such that $A' \not\vdash \neg\alpha$ and $A' \cup \{\beta\} \vdash \neg\alpha$.
(Weak disjunctive elimination) If $\alpha \in A \circledast \alpha$ and $\beta \in A \setminus (A \circledast \alpha)$, then $A \circledast \alpha \not\vdash \neg\alpha \vee \beta$.
(Weak relative extensionality) If $\alpha \in A \circledast \alpha$ and $\vdash \alpha \leftrightarrow \beta$, then $A \cap A \circledast \alpha = A \cap A \circledast \beta$.

The following observation relates some of the postulates mentioned above.

Observation 7 *Let A be a belief base and $\circledast$ be an operation on A. Then:*

1. *If $\circledast$ satisfies weak uniformity, then it satisfies weak relative extensionality.*
2. *If $\circledast$ satisfies uniform success, then it satisfies extensional success.*
3. *If $\circledast$ satisfies weak relevance, then it satisfies weak core-retainment.*
4. *If $\circledast$ satisfies weak relevance, then it satisfies weak disjunctive elimination.*
5. *If $\circledast$ satisfies weak disjunctive elimination, then it satisfies weak relative closure.*
6. *If $\circledast$ satisfies weak core-retainment, then it satisfies weak vacuity.*

The following observation shows how properties of the transformation function (eventually) combined with postulates of base revision assure the fulfilment of certain postulates of selective revision.

Observation 8 *Let A be a belief base, $*$ be a revision operator on A (i.e. an operator that satisfies success, inclusion and consistency) and f be a transformation function. Let $\circledast$ be the selective revision operator on A based on $*$ and f. Then:*

1. *If f satisfies lower boundary, then $\circledast$ satisfies stability and weak inclusion.*
2. *If f satisfies strong lower boundary, then $\circledast$ satisfies strong stability.*
3. *If f satisfies consistency preservation, then $\circledast$ satisfies consistency.*

4. *If f satisfies consistency, then $\circledast$ satisfies strong consistency.*
5. *If f satisfies inherited consistency, then $\circledast$ satisfies consistency preservation.*
6. *If f satisfies lower boundary and $*$ satisfies vacuity, then $\circledast$ satisfies weak vacuity.*
7. *If f satisfies weak maximality and $*$ satisfies vacuity, then $\circledast$ satisfies vacuity.*
8. *If f satisfies idempotence, then $\circledast$ satisfies weak proxy success.*
9. *If f satisfies idempotence and implication, then $\circledast$ satisfies proxy success.*
10. *If f satisfies lower boundary and $*$ satisfies relevance, then $\circledast$ satisfies weak relevance.*
11. *If f satisfies lower boundary and $*$ satisfies core-retainment, then $\circledast$ satisfies weak core-retainment.*
12. *If $*$ satisfies weak relative closure, then $\circledast$ satisfies weak relative closure.*
13. *If f satisfies lower boundary and $*$ satisfies disjuntive elimination, then $\circledast$ satisfies weak disjuntive elimination.*
14. *If f satisfies uniform identity and lower boundary, then $\circledast$ satisfies uniform success.*
15. *If f satisfies uniform identity and lower boundary and $*$ satisfies uniformity, then $\circledast$ satisfies weak uniformity.*
16. *If f satisfies equivalence propagation and lower boundary and $*$ satisfies relative extensionality, then $\circledast$ satisfies extensional success and weak relative extensionality.*

3.2 Representation theorems

In this subsection we present representation theorems for classes of selective base revision operators based on different base revision operators. We start by presenting a representation theorem for the most general class of selective base revision operators (that satisfies *proxy success*) that we will consider.

Theorem 9 *Let A be a belief base and $\circledast$ be an operator on A. Then the following pair of conditions are equivalent:*

(a) $\circledast$ satisfies weak inclusion, consistency, stability and proxy success.
(b) There exists a revision operator $*$ for A and a transformation function f that satisfies lower boundary, idempotence, implication and such that $A \circledast \alpha = A * f(\alpha)$, for all α.

We now present axiomatic characterizations for other less general classes of selective base revision. More precisely, we will present representation theorems for four different classes of selective revisions based, respectively, on: partial meet revisions, kernel revisions, smooth kernel revisions and basic AGM-generated base revisions.

Theorem 10 *Let A be a belief base and $\circledast$ be an operator on A. Then the following pair of conditions are equivalent:*

(a) $\circledast$ *satisfies weak inclusion, consistency, stability, proxy success, weak uniformity, uniform success and weak relevance.*
(b) *There exists a partial meet revision operator* $*$ *for* A *and a transformation function* f *that satisfies lower boundary, idempotence, implication, uniform identity and such that* $A \circledast \alpha = A * f(\alpha)$*, for all* α.

Theorem 11 *Let* A *be a belief base and* $\circledast$ *be an operator on* A*. Then the following pair of conditions are equivalent:*

(a) $\circledast$ *satisfies weak inclusion, consistency, stability, proxy success, weak uniformity, uniform success and weak core-retainment.*
(b) *There exists a kernel revision operator* $*$ *for* A *and a transformation function* f *that satisfies lower boundary, idempotence, implication, uniform identity and such that* $A \circledast \alpha = A * f(\alpha)$*, for all* α.

Theorem 12 *Let* A *be a belief base and* $\circledast$ *be an operator on* A*. Then the following pair of conditions are equivalent:*

(a) $\circledast$ *satisfies weak inclusion, consistency, stability, proxy success, weak uniformity, uniform success, weak core-retainment and weak relative closure.*
(b) *There exists a smooth kernel revision operator* $*$ *for* A *and a transformation function* f *that satisfies lower boundary, idempotence, implication, uniform identity and such that* $A \circledast \alpha = A * f(\alpha)$*, for all* α.

Theorem 13 *Let* A *be a belief base and* $\circledast$ *be an operator on* A*. Then the following pair of conditions are equivalent:*

(a) $\circledast$ *satisfies weak inclusion, consistency, stability, proxy success, weak relative extensionality, extensional success, weak vacuity and weak disjunctive elimination.*
(b) *There exists a basic AGM-generated base revision operator* $*$ *for* A *and a transformation function* f *that satisfies lower boundary, idempotence, implication, equivalence propagation and such that* $A \circledast \alpha = A * f(\alpha)$*, for all* α.

In the following definition we assign designations to the different kinds of selective revision operators that were axiomatically characterized in Theorems 10 to 13.

Definition 13 The operators characterized in Theorems 9 to 13 are respectively, basic selective base revisions, partial meet-based selective base revisions, kernel-based selective base revisions, smooth kernel-based selective base revisions and basic AGM-generated-based selective base revisions.

The following result, which is an immediate consequence of Observation 7 and Theorems 10 to 13, illustrates the interrelations among the classes of selective base revisions mentioned in the previous definition.

Observation 14 *If* $\circledast$ *is an operator of:*

1. *smooth kernel-based selective base revision, then it is also an operator of kernel-based selective base revision.*
2. *partial meet-based selective base revision, then it is also an operator of (smooth) kernel-based selective base revision and an operator of basic AGM-generated-based selective base revision*

In Figure 1 we present a diagram that summarizes the results established in Theorems 9 to 13 and in Observation 14. In that diagram an arrow between two boxes symbolizes that the class at the origin of the arrow is a subclass of the class at the end of that arrow.

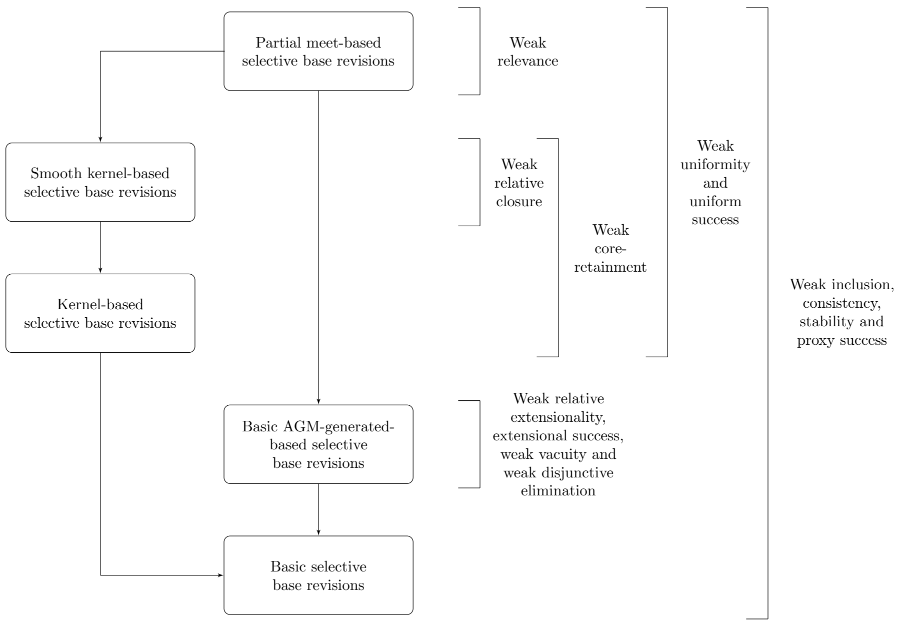

Fig. 1 Map among different classes of selective base revisions.

In the following theorem, we present five other representation theorems. More precisely, we axiomatically characterize basic selective base revisions, partial meet-based selective base revisions, (smooth) kernel-based selective base revisions and basic AGM-generated-based selective base revisions in which the associated transformation function f satisfies strong lower bounding.

Theorem 15 *Let A be a belief base and $\circledast$ be an operator on A. For each one of the Theorems 9 to 13 it holds that the condition obtained by replacing stability for strong stability in statement (a) of that theorem is equivalent to the condition obtained by replacing lower bounding by strong lower bounding in statement (b) of that theorem.*

In the next theorem, we present representation theorems similar to those presented above, but in which the operators characterized satisfy *weak proxy success* instead of *proxy success*.

Theorem 16 *Let A be a belief base and $\circledast$ be an operator on A. For each one of the Theorems 9 to 15 it holds that the condition obtained by replacing proxy success for weak proxy success in statement (a) of that theorem is equivalent to the condition obtained by replacing implication by consistency preservation in statement (b) of that theorem.*

Table 1 summarizes the representation theorems presented so far in this section.

Table 1: Schematic representation of the representation theorems presented in Theorems 9 to 16

<table>
<tr><th>There exists a</th><th>and a transformation function f that satisfies idempotence and</th><th colspan="2">such that $A \circledast \alpha = A * f(\alpha)$, for all α iff $\circledast$ satisfies weak inclusion, consistency and</th></tr>
<tr><td rowspan="4">i. partial meet revision operator/
ii. smooth kernel revision operator/
iii. kernel revision operator
$*$ for A</td><td>lower boundary, implication and uniform identity</td><td rowspan="4">weak uniformity, uniform success and
i. weak relevance/
ii. weak core-retainment and weak relative closure/
iii. weak core-retainment</td><td>stability and proxy success</td></tr>
<tr><td>strong lower boundary, implication and uniform identity</td><td>strong stability and proxy success</td></tr>
<tr><td>lower boundary, consistency preservation and uniform identity</td><td>stability and weak proxy success</td></tr>
<tr><td>strong lower boundary, consistency preservation and uniform identity</td><td>strong stability and weak proxy success</td></tr>
<tr><td rowspan="4">basic AGM-generated base revision operator
$*$ for A</td><td>lower boundary, implication and equivalence propagation</td><td rowspan="4">weak relative extensionality, extensional success, weak vacuity and weak disjunctive elimination</td><td>stability and proxy success</td></tr>
<tr><td>strong lower boundary, implication and equivalence propagation</td><td>strong stability and proxy success</td></tr>
<tr><td>lower boundary, consistency preservation and equivalence propagation</td><td>stability and weak proxy success</td></tr>
<tr><td>strong lower boundary, consistency preservation and equivalence propagation</td><td>strong stability and weak proxy success</td></tr>
<tr><td rowspan="3">revision operator
$*$ for A</td><td>lower boundary and implication</td><td rowspan="3">—</td><td>stability and proxy success</td></tr>
<tr><td>strong lower boundary and implication</td><td>strong stability and proxy success</td></tr>
<tr><td>lower boundary and consistency preservation</td><td>stability and weak proxy success</td></tr>
</table>

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	strong lower boundary and consistency preservation		strong stability and weak proxy success

In the following two theorems, we provide axiomatic characterizations for several other classes of selective base revision operators. The outcome of a selective revision by any sentence through any of the operators characterized in those theorems is consistent.[7]

Theorem 17 *Let A be a consistent belief base and $\circledast$ be an operator on A. For each one of the Theorems 9 to 16 it holds that the condition obtained by replacing consistency for strong consistency* [8] *in statement (a) of that theorem is equivalent to the condition obtained by adding inherited consistency in statement (b) of that theorem.*

Theorem 18 *Let A be a belief base and $\circledast$ be an operator on A. For each one of the Theorems 9 to 16 it holds that the condition obtained by replacing consistency for strong consistency in statement (a) of that theorem is equivalent to the condition obtained by adding consistency in statement (b) of that theorem.*[9]

4 Conclusion and future work

AGM revision operators satisfy the *success* postulate, that states that a given belief is always incorporated when revising by it. However, this is not always a desirable property of belief revision. When facing new information, a rational agent should be able to reject it or incorporate only a part of that information into his/her set of beliefs. This may happen, for example, if that information is not plausible enough or comes from unreliable sources or contradicts some of the highly entrenched beliefs of the agent.

Selective revisions were originally defined for belief sets in [6] and are operators constructed by means of a basic AGM revision and a (transformation) function f from $\mathcal{L}$ to $\mathcal{L}$. Roughly speaking, the selective revision by α is the

[7] In Theorem 17 the belief base considered must be consistent, but in Theorem 18 this condition is not required.

[8] Note that, since $A \not\vdash \perp$ it holds that *consistency preservation* and *strong consistency* are equivalent. Thus, we could also replace *strong consistency* for *consistency preservation* in the statement of this theorem.

[9] Regarding the statement of this theorem we would like to point out that if $\alpha \vdash \perp$ and $\alpha \in A$, then the consistency and lower boundary of f, are contradictory and the same occurs regarding the postulates of *stability* and *strong consistency*. Therefore statements (a) and (b) are trivially equivalent in this case. However, from a practical point of view, the most interesting case is when $A \cap \{\alpha : \alpha \vdash \perp\} = \emptyset$, where the above contradictions are not verified. This means that, by means of an appropriate selective revision it is possible to recover consistency from a non consistent original set A, provided that contradictions are not explicitly in A.

revision through a basic AGM revision by $f(\alpha)$. Intuitively, the function f returns the part of the belief that is considered to be credible and that the agent incorporates during the (selective) revision process. By imposing properties on the transformation function on which the selective revision is based, we obtain different classes of selective revisions.

In this paper we adapted the selective revision operators, that were defined for belief sets in [6], to the belief base context and presented representation theorems for several classes of selective base revision operators. In Theorem 9, we presented axiomatic characterization for the most general classes of selective base revision operators (that satisfies *proxy success*) that we have considered. In Theorems 10 to 13 we presented representation theorems for selective base revision operators based on different kinds of base revision operators, namely partial meet revisions, (smooth) kernel revisions and basic AGM-generated base revisions and presented the interrelation among the operators of those classes. The postulates used in those axiomatic characterizations follow from the ones that characterize the base revisions operators on which the selective revisions there considered are based. Therefore, the selective base revision operators presented in each one of those theorems form a broader class of operators than the one that is formed by the basic revision operators on which the former are based. In the particular case where $f(\alpha) = \alpha$ holds for all α, both types of operators coincide. Finally, we note that, regarding the classes of operators proposed in Theorems 9 to 13, we have discussed and axiomatically characterized several other variants of each one of those five classes. The representation theorems presented also allow other representation theorems to be generated if other properties are to be added to the transformation function on which selective revision is based, provided that appropriate postulates are also added. For example, adding weak maximality in statement (b) of Theorem 13 and replacing *weak vacuity* by *vacuity* in statement (a) of that theorem gives rise to a new representation theorem.

We finish this section, by presenting some potential future research topics that arise naturally in the sequence of the investigation reported in this paper:

– To define and present a representation theorem for operators of selective revision (on belief sets) based on entrenchment orders and systems of spheres. Epistemic entrenchments were introduced in [13,14] and were motivated by the principle that an agent, when forced to choose between removing one of two beliefs, will remove the less entrenched one. A selective revision (on belief sets) based on entrenchment orders should make use of some new properties of (*i.e.*, required to be fulfilled by) transformation functions and the axiomatic characterization of this type of operator might allow us to obtain counterparts for the postulates of *disjunctive inclusion* and *disjunctive overlap* (the supplementary AGM revision postulates) appropriate to the context of selective revisions. The system of spheres-based model of belief revision was proposed by Grove in [15]. A *possible world* is a maximal consistent subset of the language. Grove in [15] defined a sphere system centred on $\|\mathbf{K}\|$ (set of all possible worlds that contain $\mathbf{K}$) as an ordering

over sets of possible worlds where $\|\mathbf{K}\|$ is the innermost sphere. Figuratively, the distance between a possible world and the innermost sphere reflects its plausibility towards $\|\mathbf{K}\|$. The closer a possible world is to $\|\mathbf{K}\|$, the more plausible it is.[10] This study would offer a semantic approach to selective revisions and a more intuitive and clearer picture of some aspects of these operators.
- To define and present a representation theorem for operators of selective revision (on belief bases) based on orderings of sentences (representing beliefs). One possibility is to use ensconcement relations ([5,32]). An ensconcement can be seen as an adaptation to the belief base context of entrenchment orders. Intuitively speaking, as for entrenchement orders, a belief at a higher level of the ensconcement is more difficult to remove than one positioned at a lower level.[11]

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5 Appendix: Proofs

Lemma 1 *Let A be a belief base and $\circledast$ be a selective revision operator on A based on a revision operator $*$[12] and a transformation function f that satisfies lower boundary. If $\alpha \in A \circledast \alpha$, then $f(\alpha) = \alpha$.*

Proof Assume that $\alpha \in A \circledast \alpha$. Hence $\alpha \in A * f(\alpha)$. By $*$ *inclusion* it follows that $\alpha \in A \cup \{f(\alpha)\}$. Thus either $\alpha \in A$ or $\alpha = f(\alpha)$. It holds that f satisfies lower boundary, thus in both cases it holds that $f(\alpha) = \alpha$. □

Proof (Proof of Observation 7)
Statements 1, 2 follow trivially.
3. Assume that $\alpha \in A \circledast \alpha$. Let $\beta \in A \setminus (A \circledast \alpha)$. By *weak relevance* there exists some A' such that $A \circledast \alpha \subseteq A' \subseteq A \cup \{\alpha\}$, $A' \nvdash \bot$ but $A' \cup \{\beta\} \vdash \bot$. It holds that $\alpha \in A'$. Let $B = A' \setminus \{\alpha\}$. Hence $B \subseteq A$. On the other hand, since $A' \nvdash \bot$ it follows that $B \cup \{\alpha\} \nvdash \bot$. Thus $B \nvdash \neg\alpha$. From $A' \cup \{\beta\} \vdash \bot$ it follows that

[10] For a more in-depth approach to entrenchment orders and systems of spheres, see [8, 25].
[11] Williams in [32], defined an ensconcement relation $\preceq$ on a belief base A as a transitive and connected relation ($\alpha \preceq \beta$ or $\beta \preceq \alpha$ holds for all $\alpha, \beta \in A$) such that: i) formulae that are strictly more ensconced than a given non-tautological sentence do not (even conjointly) imply it; ii) tautologies are strictly more ensconced then non-tautological formulae; iii) two given tautologies are equally ensconced.
[12] Thus, according to Definition 3, $*$ satisfies *success, inclusion* and *consistency.*

$(B \cup \{\alpha\}) \cup \{\beta\} \vdash \perp$. Hence, by deduction $B \cup \{\beta\} \vdash \neg\alpha$.
4. Assume that $\alpha \in A \circledast \alpha$. Let $\beta \in A \setminus (A \circledast \alpha)$. Assume by *reductio ad absurdum* that $A \circledast \alpha \vdash \neg\alpha \vee \beta$. Hence $A \circledast \alpha \vdash \beta$. By *weak relevance*, there exists some A' such that $A \circledast \alpha \subseteq A' \subseteq A \cup \{\alpha\}, A' \nvdash \perp$ but $A' \cup \{\beta\} \vdash \perp$. Contradiction, since every set that contains $A \circledast \alpha$ implies β.
5. Assume that $\alpha \in A \circledast \alpha$. Let $\beta \in A \cap Cn(A \circledast \alpha \cap A)$ and assume by *reductio ad absurdum* that $\beta \notin A \circledast \alpha$. Then, by *weak disjunctive elimination*, $A \circledast \alpha \nvdash \neg\alpha \vee \beta$. On the other hand, from $\beta \in Cn(A \circledast \alpha \cap A)$ it follows by monotony of Cn that $A \circledast \alpha \vdash \beta$. Thus $A \circledast \alpha \vdash \neg\alpha \vee \beta$. Contradiction. Therefore, $\beta \in A \circledast \alpha$. Hence $A \cap Cn(A \circledast \alpha \cap A) \subseteq A \circledast \alpha$.
6. Consider that $\alpha \in A \circledast \alpha$ and $A \nvdash \neg\alpha$. Assume by *reductio ad absurdum* that $A \cup \{\alpha\} \not\subseteq A \circledast \alpha$. From $\alpha \in A \circledast \alpha$ it follows that there exists some β such that $\beta \in A$ and $\beta \notin A \circledast \alpha$. Thus, by *weak core-retainment*, there exists $A' \subseteq A$ such that $A' \nvdash \neg\alpha$ and $A' \cup \{\beta\} \vdash \neg\alpha$. Contradiction, since $A' \cup \{\beta\} \subseteq A$ and $A \nvdash \neg\alpha$. Therefore, $A \cup \{\alpha\} \subseteq A \circledast \alpha$. □

Proof (Proof of Observation 8)
1. **Stability:** Let $\alpha \in A$. f satisfies lower boundary. Thus $f(\alpha) = \alpha$. Hence $A \circledast \alpha = A * f(\alpha) = A * \alpha$. Thus, by $*$ *success* it follows that $\alpha \in A \circledast \alpha$.
Weak inclusion: Let $\alpha \in A \circledast \alpha$. Thus $f(\alpha) = \alpha$ (by Lemma 1). Hence $A \circledast \alpha = A * f(\alpha) = A * \alpha$. From which it follows by $*$ *inclusion*, that $A \circledast \alpha \subseteq A \cup \{\alpha\}$.
2. Let $A \vdash \alpha$. f satisfies strong lower boundary. Thus $f(\alpha) = \alpha$. Hence $A \circledast \alpha = A * f(\alpha) = A * \alpha$. Thus by $*$ *success* it follows that $\alpha \in A \circledast \alpha$.
3. Let $\alpha \nvdash \perp$. Hence $\nvdash \neg\alpha$. f satisfies consistency preservation. Thus $\nvdash \neg f(\alpha)$. Hence $f(\alpha) \nvdash \perp$. Therefore, by $*$ *consistency*, it follows that $A * f(\alpha) \nvdash \perp$, from which it follows by definition of $\circledast$ that $A \circledast \alpha \nvdash \perp$.
4. It holds that $\nvdash \neg f(\alpha)$. Hence $f(\alpha) \nvdash \perp$, from which it follows, by $*$ *consistency*, that $A * f(\alpha) \nvdash \perp$. Thus $A \circledast \alpha \nvdash \perp$.
5. Assume that $A \nvdash \perp$. It holds that f satisfies inherited consistency, thus $\nvdash \neg f(\alpha)$. By definition of $\circledast$ it follows that $A \circledast \alpha = A * f(\alpha)$. From which it follows by $*$ *consistency* that $A \circledast \alpha \nvdash \perp$.
6. Assume that $A \nvdash \neg\alpha$ and $\alpha \in A \circledast \alpha$. From the latter it follows, by Lemma 1, that $f(\alpha) = \alpha$. Thus $A \nvdash \neg f(\alpha)$. By $*$ *vacuity* it follows that $A * f(\alpha) \subseteq A \cup \{f(\alpha)\}$. Thus $A \circledast \alpha \subseteq A \cup \{\alpha\}$.
7. Assume that $A \nvdash \neg\alpha$. f satisfies weak maximality, thus $f(\alpha) = \alpha$. Hence, by $*$ *vacuity*, it follows that $A \cup \{\alpha\} \subseteq A * \alpha = A * f(\alpha) = A \circledast \alpha$.
8. It holds that $A \circledast \alpha = A * f(\alpha)$. By $*$ *success* it follows that $f(\alpha) \in A * f(\alpha) = A \circledast \alpha$. On the other hand, f satisfies idempotence, thus $f(f(\alpha)) = f(\alpha)$. Therefore, $A * f(\alpha) = A * f(f(\alpha))$. By definition of $\circledast$ it follows that $A * f(f(\alpha)) = A \circledast f(\alpha)$. Hence $f(\alpha) \in A \circledast \alpha = A \circledast f(\alpha)$.
9. As showed in the previous item, it holds that $f(\alpha) \in A \circledast \alpha = A \circledast f(\alpha)$. On the other hand, f satisfies implication, thus $\vdash \alpha \to f(\alpha)$.
10. Let $\alpha \in A \circledast \alpha$. By Lemma 1 it follows that $f(\alpha) = \alpha$. Hence $A \circledast \alpha = A * \alpha$. Let $\beta \in A$ and $\beta \notin A \circledast \alpha$. Thus, by $*$ *relevance*, it follows that there is some A' such that $A \circledast \alpha \subseteq A' \subseteq A \cup \{\alpha\}, A' \nvdash \perp$ but $A' \cup \{\beta\} \vdash \perp$.

11. The proof that $\circledast$ satisfies *weak core-retainment* is similar to the proof presented for *weak relevance*.
12. Assume that $*$ satisfies weak relative closure. It holds by definition of $\circledast$ that $A \cap Cn(A \cap A \circledast \alpha) = A \cap Cn(A \cap A * f(\alpha))$. By $*$ *weak relative closure* it holds that $A \cap Cn(A \cap A * f(\alpha)) \subseteq A * f(\alpha)$. From which it follows, by definition of $\circledast$, that $A \cap Cn(A \cap A \circledast \alpha) \subseteq A \circledast \alpha$.
13. Let $\alpha \in A \circledast \alpha$ and $\beta \in A \setminus (A \circledast \alpha)$. From the former it follows by Lemma 1 that $f(\alpha) = \alpha$ and from the latter that $\beta \in A \setminus (A * f(\alpha))$. By $*$ *disjunctive elimination* it follows that $A * f(\alpha) \not\vdash \neg f(\alpha) \vee \beta$. Hence $A \circledast \alpha \not\vdash \neg f(\alpha) \vee \beta$. Therefore, $A \circledast \alpha \not\vdash \neg\alpha \vee \beta$.
14. Consider that it holds for all subsets A' of A that $A' \cup \{\alpha\} \vdash\perp$ if and only if $A' \cup \{\beta\} \vdash\perp$. Assume that $\alpha \in A \circledast \alpha$. By Lemma 1 it follows that $f(\alpha) = \alpha$. On the other hand, f satisfies uniform identity, thus $f(\beta) = \beta$. By definition of $\circledast$ it holds that $A \circledast \beta = A * f(\beta)$. By $*$ *success* it follows that $f(\beta) \in A \circledast \beta$. Thus $\beta \in A \circledast \beta$. By symmetry of the case it holds that if $\beta \in A \circledast \beta$, then $\alpha \in A \circledast \alpha$. Hence it holds that $\alpha \in A \circledast \alpha$ if and only if $\beta \in A \circledast \beta$.
15. Let $\alpha \in A \circledast \alpha$. Consider that it holds for all subsets A' of A that $A' \cup \{\alpha\} \vdash\perp$ if and only if $A' \cup \{\beta\} \vdash\perp$. It holds that $\circledast$ satisfies *uniform success* (as showed in the previous item), thus $\beta \in A \circledast \beta$. On the other hand, from $\alpha \in A \circledast \alpha$ it follows that $f(\alpha) = \alpha$ (by Lemma 1). By symmetry of the case it follows that $f(\beta) = \beta$. On the other hand, by $*$ *uniformity*, it follows that $A \cap A * \alpha = A \cap A * \beta$. Thus $A \cap A \circledast \alpha = A \cap A * f(\alpha) = A \cap A * \alpha = A \cap A * \beta = A \cap A * f(\beta) = A \cap A \circledast \beta$.
16. Let $\vdash \alpha \leftrightarrow \beta$. Assume that $\alpha \in A \circledast \alpha$. From $\alpha \in A \circledast \alpha$ it follows that $f(\alpha) = \alpha$ (by Lemma 1). It holds that f satisfies equivalence propagation. Thus $f(\beta) = \beta$. On the other hand, by definition of $\circledast$, it follows that $A \circledast \beta = A * f(\beta)$. By $*$ *success* it holds that $f(\beta) \in A * f(\beta)$, from which it follows that $\beta \in A \circledast \beta$. Hence:
- **Extensional success:** By symmetry of the case, if $\beta \in A \circledast \beta$ then $\alpha \in A \circledast \alpha$. Thus $\alpha \in A \circledast \alpha$ holds if and only if $\beta \in A \circledast \beta$ holds.
- **Weak relative extensionality:** From $\beta \in A \circledast \beta$ it follows, by Lemma 1, that $f(\beta) = \beta$. On the other hand by $*$ *relative extensionality* it follows that $A \cap A * \alpha = A \cap A * \beta$. Thus $A \cap A \circledast \alpha = A \cap A * f(\alpha) = A \cap A * \alpha = A \cap A * \beta = A \cap A * f(\beta) = A \cap A \circledast \beta$. □

Proof (Proof of Theorem 9)
(a) implies (b): We will first define f and $*$. Let $f : \mathcal{L} \longrightarrow \mathcal{L}$ be such that:

$$f(\alpha) = \begin{cases} \alpha & \text{if } \alpha \in A \circledast \alpha \\ r(\alpha) & \text{otherwise} \\ \text{where } r \text{ is a function from } \mathcal{L} \text{ to } \mathcal{L} \text{ such that} & \\ r(\alpha) \in A \circledast \alpha, A \circledast \alpha = A \circledast r(\alpha) & \\ \text{ and } \vdash \alpha \rightarrow r(\alpha). & \end{cases}$$

Let

$$A * \alpha = \begin{cases} A \circledast \alpha & \text{if } \alpha \in A \circledast \alpha \\ A *' \alpha & \text{otherwise} \\ \text{where } *' \text{ is any revision operator.} & \end{cases}$$

We need to show that:
(i) f is a (well defined) function that satisfies the properties listed in (b);
(ii) $*$ satisfies *inclusion, success* and *consistency*;
(iii) $A \circledast \alpha = A * f(\alpha)$, for all α.
(i) To prove that f is a (well defined) function we must show that for all $\alpha \in \mathcal{L}$ there exists $\alpha' \in \mathcal{L}$ such that $f(\alpha) = \alpha'$ and that, if $\alpha_1 = \alpha_2$, then $f(\alpha_1) = f(\alpha_2)$. Let $\alpha \in \mathcal{L}$. If $\alpha \in A \circledast \alpha$, then $f(\alpha) = \alpha$. If $\alpha \notin A \circledast \alpha$, then $f(\alpha) = r(\alpha) = \alpha'$, for some α' such that $\alpha' \in A \circledast \alpha = A \circledast \alpha'$ and $\vdash \alpha \to \alpha'$. Such α' exists since $\circledast$ satisfies *proxy success*. Assume now that $\alpha_1 = \alpha_2$. If $\alpha_1 \in A \circledast \alpha_1$, then $\alpha_2 \in A \circledast \alpha_2$. Thus $f(\alpha_1) = \alpha_1 = \alpha_2 = f(\alpha_2)$. If $\alpha_1 \notin A \circledast \alpha_1$, then $\alpha_2 \notin A \circledast \alpha_2$. Thus $f(\alpha_1) = r(\alpha_1)$ and $f(\alpha_2) = r(\alpha_2)$. r is a function. Thus, from $\alpha_1 = \alpha_2$ it follows that $f(\alpha_1) = f(\alpha_2)$.
That f satisfies lower boundary follows by definition of f and $\circledast$ *stability*.
That f satisfies implication follows by definition of f. To show that f satisfies idempotence:
case 1) $\alpha \in A \circledast \alpha$. Thus $f(\alpha) = \alpha$. Hence $f(f(\alpha)) = f(\alpha)$.
case 2) $\alpha \notin A \circledast \alpha$. Thus $f(\alpha) = r(\alpha)$ and $r(\alpha) \in A \circledast r(\alpha)$. Hence, by definition of f, it follows that $f(r(\alpha)) = r(\alpha)$. From the latter and $f(\alpha) = r(\alpha)$ it follows that $f(f(\alpha)) = f(\alpha)$.
(ii) That $*$ satisfies *success* follows trivially by definition of $*$. That $*$ satisfies *consistency* follows from the fact that both $\circledast$ and $*'$ satisfy *consistency*. Next we show that $*$ satisfies *inclusion*. If $\alpha \in A \circledast \alpha$, then by $\circledast$ *weak inclusion*, it follows that $A \circledast \alpha \subseteq A \cup \{\alpha\}$. From which it follows, by definition of $*$, that $A * \alpha \subseteq A \cup \{\alpha\}$. If $\alpha \notin A \circledast \alpha$, then $A * \alpha = A *' \alpha$. It holds that $*'$ satisfies *inclusion* thus $A * \alpha \subseteq A \cup \{\alpha\}$.
(iii) We will now prove that $A \circledast \alpha = A * f(\alpha)$.
case 1) $\alpha \notin A \circledast \alpha$. By definition of f it holds that $A \circledast \alpha = A \circledast f(\alpha)$ and $f(\alpha) \in A \circledast \alpha$. Hence $f(\alpha) \in A \circledast f(\alpha)$. Thus, by definition of $*$, it follows that $A * f(\alpha) = A \circledast f(\alpha)$, from which it follows that $A \circledast \alpha = A * f(\alpha)$.
case 2) $\alpha \in A \circledast \alpha$. Hence $f(\alpha) = \alpha$ and $A * \alpha = A \circledast \alpha$. Thus $A * f(\alpha) = A \circledast \alpha$.
(b) implies (a): It holds that f satisfies implication, thus it also satisfies consistency preservation. That $\circledast$ satisfies the postulates listed in condition (a) of the theorem follows from Observation 8. □

Proof (Proof of Theorem 10)
(a) implies (b): We will use essentially, for this part of the proof, the same constructions as in the proof of Theorem 9. The only difference is that, in this proof, the operator $*'$ used in the definition of $*$ is a partial meet revision instead of a revision operator as in the proof of former theorem. We need to show that:
(i) f is a (well-defined) function that satisfies the properties listed in (b);
(ii) $*$ satisfies *inclusion, success, consistency, uniformity* and *relevance* (by Observation 3);
(iii) $A \circledast \alpha = A * f(\alpha)$, for all α.
It holds that $*'$ is a partial meet revision. Thus, according to Observation 3 it satisfies *inclusion, success, consistency, uniformity* and *relevance*.

(i) The proof that f is a (well-defined) function that satisfies lower boundary, idempotence and implication is similar to the one presented for Theorem 9. We will now show that f satisfies uniform identity. Assume that for all subsets $A' \subseteq A$ it holds that $A' \cup \{\alpha\} \vdash \bot$ if and only if $A' \cup \{\beta\} \vdash \bot$. Let $f(\alpha) = \alpha$. By definition of f it holds that $f(\alpha) \in A \circledast \alpha$. Thus $\alpha \in A \circledast \alpha$. By $\circledast$ *uniform success* it follows that $\beta \in A \circledast \beta$. Thus $f(\beta) = \beta$. By symmetry of the case it follows that if $f(\beta) = \beta$, then $f(\alpha) = \alpha$. Hence it holds that $f(\alpha) = \alpha$ if and only if $f(\beta) = \beta$.
(ii) That $*$ satisfies *success, consistency* and *inclusion* follows as in the proof of Theorem 9. That $*$ satisfies *relevance* follows from the definition of $*$, $\circledast$ *weak relevance* and $*'$ *relevance*. To prove that $*$ satisfies *uniformity* assume that for all subsets $A' \subseteq A$, it holds that $A' \cup \{\alpha\} \vdash \bot$ if and only if $A' \cup \{\beta\} \vdash \bot$. By $\circledast$ *uniform success* it follows that $\alpha \in A \circledast \alpha$ if and only if $\beta \in A \circledast \beta$. We will consider two cases:
case 1) $\alpha \in A \circledast \alpha$. Then $\beta \in A \circledast \beta$. Thus $A * \alpha = A \circledast \alpha$ and $A * \beta = A \circledast \beta$. From which it follows by $\circledast$ *weak uniformity* that $A \cap A * \alpha = A \cap A * \beta$.
case 2) $\alpha \notin A \circledast \alpha$. Then $\beta \notin A \circledast \beta$. Thus $A * \alpha = A *' \alpha$ and $A * \beta = A *' \beta$. Hence by $*'$ *uniformity* it follows that $A \cap A * \alpha = A \cap A * \beta$.
(iii) This follows as in the proof of Theorem 9.
(b) implies (a): It holds that f satisfies implication, thus it also satisfies consistency preservation. That $\circledast$ satisfies the postulates listed in condition (a) of the theorem follows from Observation 8. □

Proof (Proof of Theorem 11)
(a) implies (b): We will use essentially, for this part of the proof, the same constructions as in the proof of Theorem 9. The only difference is that, in this proof, the operator $*'$ used in the definition of $*$ is a kernel revision instead of a revision operator as in the proof of that theorem (hence, according to Observation 4, $*'$ satisfies *inclusion, success, consistency, uniformity* and *core-retainment*). The proof, for this part, is very similar to the one presented for Theorem 10. The only difference is that we need to show that $*$ satisfies *core-retainment* instead of *relevance*. That $*$ satisfies *core-retainment* follows from the definition of $*$, $\circledast$ *weak core-retainment* and $*'$ *core-retainment*.
(b) implies (a): It holds that f satisfies implication, thus it also satisfies consistency preservation. That $\circledast$ satisfies the postulates listed in condition (a) of the theorem follows from Observation 8. □

Proof (Proof of Theorem 12)
(a) implies (b): We will use essentially, for this part of the proof, the same constructions as in the proof of Theorem 9 but in this case the operator $*'$, used in the definition of $*$, is a smooth kernel revision. The proof, for this part, is very similar to the one presented for Theorem 11. The only difference is that additionally we need to show that $*$ satisfies *weak relative closure*. This follows from the definition of $*$ since $\circledast$ and $*'$ satisfy *weak relative closure*.
(b) implies (a): It holds that f satisfies implication, thus it also satisfies consistency preservation. That $\circledast$ satisfies the postulates listed in condition (a) of the theorem follows from Observation 8. □

Proof (Proof of Theorem 13)
(a) implies (b): Once more, for this part of the proof, we will use a similar construction to the one used in the proof of Theorem 9 but in this case the operator $*'$, in the definition of $*$, is a basic AGM-generated base revision operator instead of a revision operator as in the proof of that theorem. We need to show that:
(i) f is a (well-defined) function that satisfies the properties listed in (b);
(ii) $*$ satisfies *success, consistency, inclusion, vacuity, relative extensionality* and *disjunctive elimination* (by Observation 6);
(iii) $A \circledast \alpha = A * f(\alpha)$, for all α.
It holds that $*'$ is a basic AGM-generated base revision. Thus, according to Observation 6 it satisfies *success, consistency, inclusion, vacuity, relative extensionality* and *disjunctive elimination.*
(i) The proof that f is a (well-defined) function that satisfies lower boundary, idempotence and implication is similar to the one presented for Theorem 9. It remains to show that f satisfies equivalence propagation.
Assume that $\vdash \alpha \leftrightarrow \beta$. Let $f(\alpha) = \alpha$. By definition of f it holds that $f(\alpha) \in A \circledast \alpha$. Thus $\alpha \in A \circledast \alpha$. By $\circledast$ *extensional success* it follows that $\beta \in A \circledast \beta$. From which it follows that $f(\beta) = \beta$. By symmetry of the case it follows that if $f(\beta) = \beta$, then $f(\alpha) = \alpha$. Hence it holds that $f(\alpha) = \alpha$ if and only if $f(\beta) = \beta$.
(ii) The proof that $*$ satisfies *success, consistency* and *inclusion* follows as in the proof of Theorem 9.
Vacuity: Follows from the definition of $*$, $\circledast$ *weak vacuity* and $*'$ *vacuity.*
Disjunctive elimination: Follows from the definition of $*$, $\circledast$ *weak disjunctive elimination* and $*'$ *disjunctive elimination.*
Relative extensionality: Assume that $\vdash \alpha \leftrightarrow \beta$. By $\circledast$ *extensional success*, $\alpha \in A \circledast \alpha$ holds if and only if $\beta \in A \circledast \beta$ holds. We will consider two cases:
case 1) $\alpha \in A \circledast \alpha$. Then $\beta \in A \circledast \beta$. Thus $A * \alpha = A \circledast \alpha$ and $A * \beta = A \circledast \beta$. Hence, by $\circledast$ *weak relative extensionality*, it follows that $A \cap A * \alpha = A \cap A * \beta$.
case 2) $\alpha \notin A \circledast \alpha$. Then $\beta \notin A \circledast \beta$. Thus $A * \alpha = A *' \alpha$ and $A * \beta = A *' \beta$. Hence by $*'$ *relative extensionality* it follows that $A \cap A * \alpha = A \cap A * \beta$.
(iii) This follows as in the proof of Theorem 9.
(b) implies (a): It holds that f satisfies implication, thus it also satisfies consistency preservation. That $\circledast$ satisfies the postulates listed in condition (a) of the theorem follows from Observation 8. □

Proof (Proof of Theorem15)
From (b) to (a) follows by Observation 8, since strong lower bounding implies lower bounding and implication implies consistency preservation. From (a) to (b) we use the same construction as in the proofs of Theorems 9 to 13. Since *strong stability* implies *stability* we only need to prove that the transformation function f satisfies strong lower bounding. This follows by definition of f and by $\circledast$ *strong stability.* □

Proof (Proof of Theorem16)
From (b) to (a) follows by Observation 8. From (a) to (b) we use essentially

the same construction as in the proofs of Theorems 9 to 15 removing the condition $\vdash \alpha \to r(\alpha)$ in the definition of f. The existence of such function follows from *weak proxy success*. The proof is similar to the ones presented for those theorems, the main difference is the proof that f satisfies consistency preservation. Assume that $\nvdash \neg\alpha$. If $\alpha \in A \circledast \alpha$, then $f(\alpha) = \alpha$, from which it follows that $\nvdash \neg f(\alpha)$. Consider now that $\alpha \notin A \circledast \alpha$. By definition of f it follows that $f(\alpha) \in A \circledast \alpha$, from which it follows, by $\circledast$ *consistency*, that $A \circledast \alpha \nvdash \bot$. Therefore $f(\alpha) \nvdash \bot$. Hence $\nvdash \neg f(\alpha)$. □

Proof (Proof of Theorem17)
From (b) to (a) follows by Observation 8 (having in mind that if f satisfies implication, then it also satisfies consistency preservation). From (a) to (b) we use the same construction as in the proofs of Theorems 9 to 16. We only need to prove that f satisfies inherited consistency. This follows by definition of f and by $\circledast$ *strong consistency*. □

Proof (Proof of Theorem18)
From (b) to (a) follows by Observation 8 (having in mind that if f satisfies implication, then it also satisfies consistency preservation). From (a) to (b) we will use the same construction as in the proofs of Theorems 9 to 16. It holds that $\circledast$ satisfies *consistency* (since it satisfies *strong consistency*). Thus, we only need to prove that f satisfies consistency. This follows by definition of f and by $\circledast$ *strong consistency*. □

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