Available online at www.sciencedirect.com

ScienceDirect

Procedia Structural Integrity 28 (2020) 279–286

Structural Integrity Procedia

www.elsevier.com/locate/procedia

1st Virtual European Conference on Fracture

Direct Integration Methods versus Modal Superposition Method, on Predicting Staircases Vibrations

Pedro Andrade[a], José Santos[b,c,*], Patrícia Escórcio[b]

[a]*University of Madeira, 9020-105 Funchal, Portugal*
[b]*University of Madeira, Faculty of Exact Sciences and Engineering, Department of Civil Engineering and Geology, 9020-105 Funchal, Portugal*
[c]*CONSTRUCT-LABEST, Faculty of Engineering (FEUP), University of Porto, Portugal*

Abstract

The majority of Finite Element software's present two different solutions methods to perform time history analysis of the equations of motion due to dynamic (time-varying) loads: Direct Integration and Modal Superposition. This paper aims to assess which method should be employed in the design of modern flexible staircases, to more efficiently predict human induced vibrations. This was verified by estimating vibrations on a real staircase using the two time domain analysis methods and, then, comparing with vibrations experimentally measured. The results indicate that Direct Integration could yield to overestimated responses due to the limited capacity, as the vibration modes increase, of FE numerical models to realistic predict natural frequencies and modal shapes of a real structure. Therefore, Modal Superposition is suggested to be used for design routines, excluding, for the same reason, the vibration modes with higher frequency content.

Keywords: Modal Superposition; Direct Integration; Duhamel Integral; Human Walking Vibrations; Flexible Staircases.

* Corresponding author. Tel.: +351 291 705 197.
E-mail address: jmmns@fe.up.pt

10.1016/j.prostr.2020.10.033

1. Introduction

In recent years, it is becoming a trend in contemporary architecture to design slender staircases, with fewer supports and longer spans, which are significantly more flexible and vibration-prone when compared to robust traditional design. Nowadays, the use of Finite Element (FE) commercial software's for the verification of ultimate limit states (ULS), in the designing of structures to static loads, is relatively trivial. However, despite significant advances in numerical prediction, using FE models when designing flexible staircases to be insensitive to vibrations due human walking (serviceability limit states (SLS)), is still a complex challenge. Predicting vibration susceptibility of staircases within an FE software requires the use of dynamic analysis that are not yet fully developed, have some limitations and are not well understood by the majority of structural designers. With the increasing popularity of building slender and lighter solutions, the deepening of designer's understanding of dynamic analysis methods using FE software's is becoming more and more important.

Most of FE software's currently present two methods of dynamic analysis in time domain that can be used to numerically calculate human induced vibrations, Direct Integration and Modal Superposition. Modal Superposition involves decoupling the system equations of motion, solving the uncoupled equations, and re-coupling, to compute responses for a different number of natural frequencies and shapes of the structure's vibration modes. This allows the user control over which modes are included in the response history analysis and serves as a low pass filter, excluding frequency content above the frequency of the highest computed mode. Direct integration of the coupled equations of motion computes structure's dynamic response, offering no control over which modes are considered (Davis, 2008).

In this paper it is discussed which is the most efficient and, therefore, should be employed when designing flexible staircases. This is achieved by first experimentally measuring the accelerations due to human walking on a steel staircase whose vibration level is significant. Then, a very detailed numerical model of the steel staircase is elaborated using the FE software SAP2000, where several dynamic analysis are performed using Direct Integration and Modal Superposition, in order to compare the accelerations calculated numerically and those experimentally measured.

2. Experimental campaign

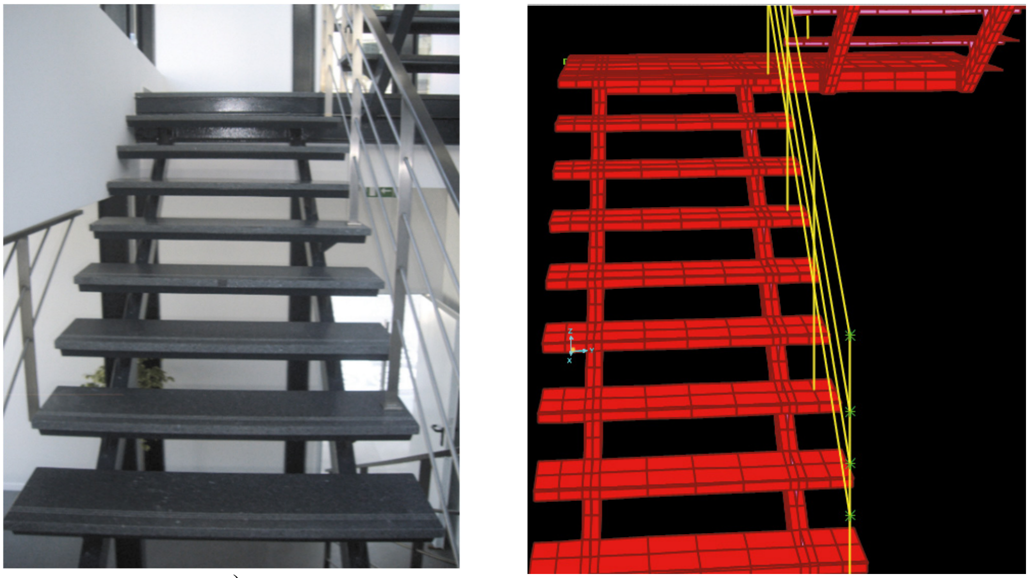
2.1. Staircase description and dynamic characterization

The steel staircase analysed in this study is located in Funchal, Madeira, Portugal. This particular staircase exhibits a high level of liveness, raising to its users discomfort and the feeling that the structure is not safe. It connects the three floors of the building with its identical four flights of steps. Each flight of steps, represented in Fig. 1a), is constituted by two stringers with a steel hollow structural section (HSS) 120x60x4 mm and steps having a length of 1.15 m and a width of 0.32 m, being supported on the building floors by a European wide flange beam HEB180 and in the intermediated landings by three columns, also made of European wide flange beams HEB180. The total span between supports is 4.44 m. The stair steps and intermediate landings are composed of a 3 mm thick metal plate coated by a granite sheet stone of 30 mm thick (see Fig. 1a)). The connection between HSS 120x60x4 mm stringers and the HEB180 beams of each floor is made by means of an 8 mm metal plate and an M 20x100 mm screw, which makes rotational movement possible. Hence, the support could be assumed as pinned with the behavior of the two upper flights being independent of the two lower flights.

An ambient modal analysis was performed to determine the steel staircase dynamic properties. The natural frequencies and corresponding modal shapes represented in Table 1 were obtained by recording accelerations in free vibration near to the driving point and other locations of interest and, subsequently, calculating using a specifically program created on MATLAB. The damping was consistently estimated to be 1.18 % of the critical using the half-power bandwidth method, which is in agreement with the measurements on steel staircases of researchers Bishop et al. (1995), Davis et al. (2015; 2009), González (2013) and Andrade et al. (2017; 2017b), who obtained values of approximately 1 %.

2.2. Walking tests

The sample staircase, as can be observed from Table 1, has a fundamental frequency equal to 13.9 Hz, which is lower than the cut-off frequency of 16 Hz suggested by Andrade et al. (2017a) and Santos et al. (2019) and, therefore, with a possibility of resonance effects to occur. Considering that pedestrians walk on staircases with step frequencies ranging approximately from 2.0 to 4.5 Hz (Bishop et al. (1995), Davis et al. (2009) and Kasperski & Czwikla (2012)), ascending and descending at 3.5 Hz (4^{th} sub-multiple of the staircase fundamental) could generate a resonant build-up and amplify the staircase response. However, the objective of this study is to compare the Direct Integration and Modal Superposition when computing human induced vibrations, the two different time domain analysis methods being comparable in the same basis, regardless the staircase responses. Hence, walking tests and vibration measurements were performed for step frequencies more commonly used by pedestrians in their daily routines, i.e. ascents at 2.0 Hz.



a) b)

Fig. 1. Flight of steps: (a) real staircase; (b) FE staircase model.

Table 1 – Experimental and numerical vibration modes.

Modes		Experimental	Numerical
Nº	Shape	Frequency (Hz)	Frequency (Hz)
1	Vertical	13.9	13.9
2	Vertical	14.5	14.9
3	Torsion	20.9	23.4
4	Torsion	21.9	26.5
5	Torsion	22.4	27.1
6	Torsion	23.2	29.3

3. Numerical analysis

3.1. Model and dynamic properties

The FE software SAP2000 was used to compare the Direct Integration and Modal Superposition. Before comparing the two time domain numerical methods, a very detailed FE model of the studied staircase was created, calibrating its dynamic properties with the real structure, in order to the numerical results calculated by the two methods could be realistically compared with the experimental results. All the structural elements described in Subsection 2.1 were modelled using shell elements, only being considered frame elements in the modelling of the guardrails. To the shell and beam elements were attributed the mechanical properties of the materials employed in its construction, i.e. steel S275 and granite. It is important to note that due the aforementioned reasons explain in Subsection 2.1, only the two upper flight of steps were modelled and considered in the numerical analysis. Fig. 1b) represents the FE model of a flight of steps. The vibrations modes and respective frequencies were predicted using the Eigen Vectors analysis option presented in SAP2000. Table 1 presents the first six modes numerically obtained.

3.2. Direct Integration and Modal Superposition

Theoretically, direct integration is the more scientifically accurate method for numerically calculating accelerations, since it takes into account all the structure's vibration modes and higher the number of modes considered, presumably, more realistic are the numerical results. However, according to Davis (2008) and Barret (2006), as the number of modes increase, FE models have a limited capacity to successfully predict the frequencies and modal shapes of the real structure. Therefore, using direct integration to calculate accelerations can lead to erroneous and overestimated values, since the response takes into account the contribution of all structure's vibration modes and numerous may not be comparable with the real structure. Comparing the experimentally measured and numerical predicted vibrations modes seen in Table 1 furthermore corroborates this observation. The first and second vibrations modes were closely predicted, but as the number of modes increase, the difference between numerical and experimental values becomes higher.

Moreover, mentioning Davis (2008), the structure's response will be mostly conditioned by low frequency modes, as these are within the frequency range that is excitable by the human walking. Thus, the use of Modal Superposition may be an advantage over direct integration, since it allows having control over the number of modes considered, filtering out the contribution of high frequency content modes that are not be of interest.

In the following Section, different analysis are performed to further develop this researcher work and assess which numerical method to date, Direct Integration or Modal Superposition, should more accurately employed when designing flexible staircases with human induced vibrations in mind. This was achieved by evaluating the methods in two distinct scenarios: computing and comparing vibrations on a single degree of freedom (SDOF) simply supported beam and on a multi degree of freedom system (MDOF), the sample staircase.

4. Comparison between time domain numerical methods

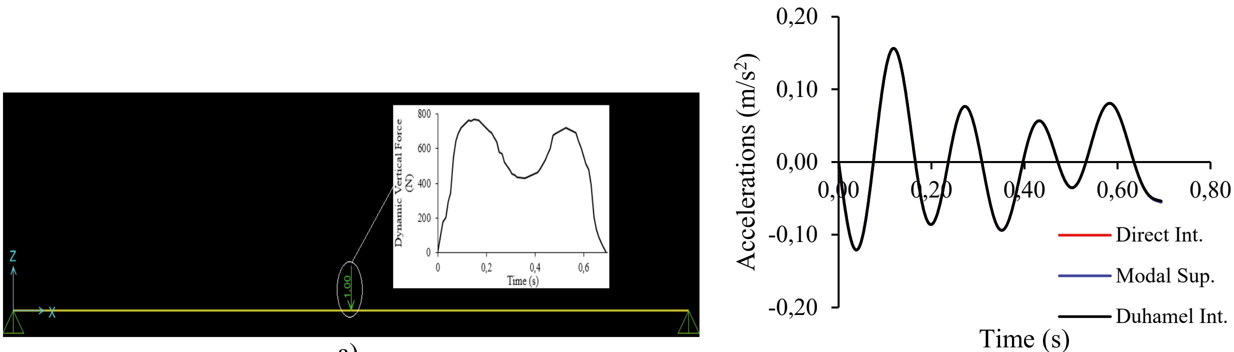
4.1. Application to an SDOF model (simply supported beam)

With the aim to evaluate which method is most suitable to employ in practice, first the accelerations were calculated through Modal Superposition and Direct Integration on an SDOF simply supported beam. An arbitrary concrete beam with a cross-section of 40 cm height and 30 cm width and a length of 10 m was considered in the numerical analysis. Using a damping value of 5 % of critical, typical for concrete structures, it was estimated a fundamental frequency equal to 6.50 Hz. Since the SDOF model response is governed by its fundamental frequency, the numerical accelerations were also calculated by the Duhamel's integral and compared with the Modal Superposition and Direct Integration for validation. The accelerations are obtained by deriving twice the Equation (1) that defines the Duhamel integral for structures with damping.

$$u(t) = \frac{1}{mw_d}\int_0^t p(\tau)e^{-\xi w_d(t-\tau)} senw_d(t-\tau)d\tau \tag{1}$$

Where $u(t)$ is the displacement, m the mass and ξ the damping coefficient of the structure, w_d is the angular frequency of the damped structure, t is the instant at which the response $u(t)$ is being calculated and τ is the instant that load $p(t)$ is acting and the variable that is integrated ($\tau < t$).

The SDOF model of the simply supported beam is represented in 2a). The numerical accelerations from the three different methods were obtained by the application at midspan of a ground reaction force (GRFs) for an ascent at 2.0 Hz directly measured in stairs by González (2013; 2014), as also shown in 2a). After applying the walking load due to a footfall at the beam midspan, the resulting accelerations for the three numerical methods are presented in the 2b). As can be seen, when dealing with an SDOF structure where the response is conditioned by its fundamental frequency, the results obtained by the Duhamel's integral, Modal Superposition and Direct Integration are virtually coincident.



a) b)

Fig. 2. SDOF simply supported beam: (a) GRF trace application; (b) Numerical accelerations.

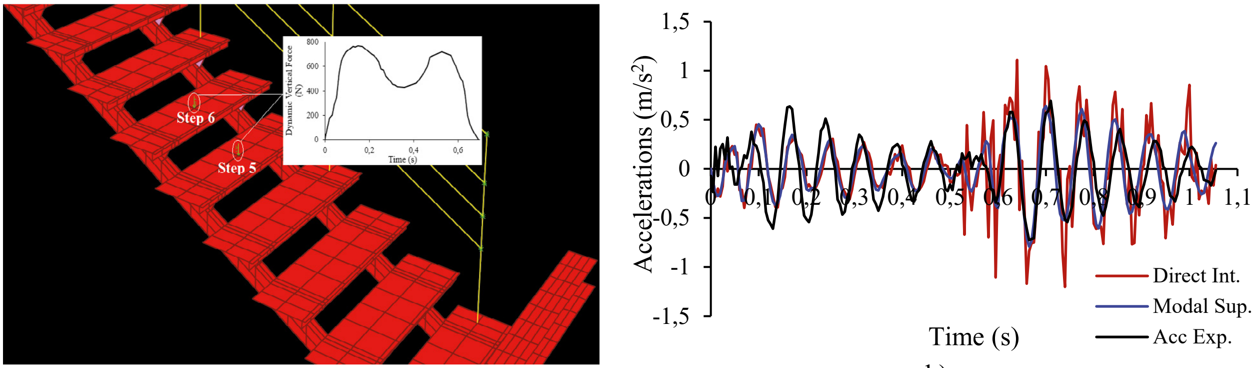
4.2. Application to an MDOF model (FE staircase model)

4.2.1 Direct Integration versus Modal Superposition "Classical" versus Experimental

In this Subsection, after comparing the accelerations in an SDOF system obtained by Direct Integration and Modal Superposition, the two time domain methods are used to calculate the numerical response of the studied staircase, a multi degree of freedom (MDOF) system. The Duhamel's integral was not used in this model, since it is an MDOF.

The same GRF for an ascent at 2.0 Hz used in the SDOF simply supported beam was applied on two consecutive steps located at midspan of the FE staircase model, step 5 and 6, as shown in 3a). Then, two separate time history analysis with the Direct Integration and Modal Superposition were performed in SAP2000, to compute the numerical accelerations of the sample staircase. In the definition of the time history analysis was considered the damping coefficient experimentally measured of 1.18 %.

As in the SDOF model, the accelerations obtained through these two methods are also graphically compared, as represented in 3b). Additionally, the accelerations measured for an ascent at 2.0 Hz from the walking tests described in the Subsection 2.2 are also included in the graph of 3b), to verify the precision of both numerical methods. 3b) represents the accelerations obtained on Step 6.



a) b)

Fig. 3. MDOF sample steel staircase: (a) GRFs traces application in the FE model; (b) Numerical and experimental accelerations.

Comparing the numerical accelerations calculated with the Direct Integration and the Modal Superposition in the FE staircase model, mainly when applying the GRF on the Step 6 (same location of response and applied load (from 0.5 s)), it can be observed that in an MDOF structure, where the response is governed by the frequencies of a higher number of vibration modes, the results are no longer coincident. The Direct Integration generated overestimated accelerations, reaching values approximately twice higher than the Modal Superposition and experimentally measured. In contrast, the accelerations obtained by the Modal Superposition are very close to the experimental accelerations during the time the two footfalls forces are applied.

This furthermore correlates with Davis (2008) and Barret (2006) observations that, due the limited capacity of FE models effectively predict the vibration modes of the real structure, having no control over the number of modes considered will probably give rise to unrealistic results.

4.2.2 Analysis of the number of vibration modes on the Modal Superposition

Although, apparently, Modal Superposition is currently the most suitable numerical method, another question arises concerning the number of vibration modes that must be employed in the accelerations calculation. The use of a large number of modes can lead to the occurrence of overly high accelerations, as in the case of the Direct Integration. In order to understand how many vibrations modes should be considered in the Modal Superposition, the accelerations were repeatedly calculated for different number of vibrations modes and compared with the experimental accelerations. For these analysis, numerical and experimental accelerations were compared for a step frequency of 2.0 Hz, analogous to the previous Subsection. The comparison with the experimental accelerations served as a reference in relation to the number of modes that should be considered.

Accelerations have been initially calculated for 100 modes, progressively reducing in identical intervals of vibration modes, applying for consistency the GFR trace for an ascent at 2.0 Hz seen in Figs. 2a) and 3a) and performing time histories analysis with a 1.18 % damping coefficient. Contrary to the previous Subsection, the accelerations were obtained by applying the GRF trace along the flight of steps, rigorously simulating the pedestrian walking during the tests described in Subsection 2.2, for subsequent comparison with the experimental results.

Fig. 4a) represents the graph of the accelerations obtained for 100 vibration modes, being possible to observe that for this number of modes the numerical accelerations are substantially higher than the experimental accelerations. As the number of modes decreases, the accelerations also decrease getting closer to the experimental accelerations, culminating in a higher level of approximation when considering a number of vibration modes equal to 10. The comparison between accelerations obtained for 10 modes and experimentally measured can be seen in Fig. 4b).

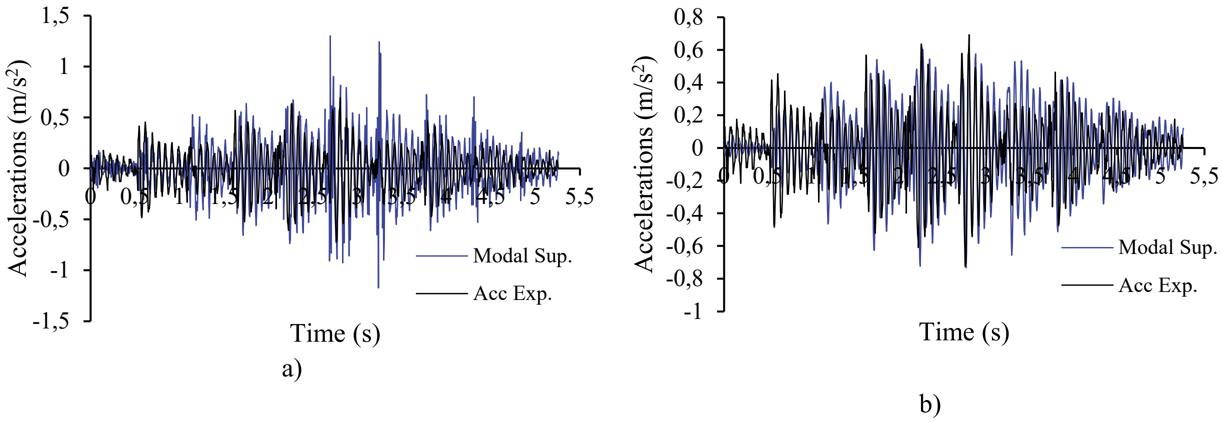


Fig. 4. Numerical and experimental accelerations: (a) 100 vibration modes; (b) 10 vibration modes.

For a clearer interpretation of the previously described, Fig. 5 illustrates the variation of peak accelerations with the number of modes and comparison with the experimental peak acceleration at 2.0 Hz. The experimental peak acceleration being plotted as a reference.

When comparing the numerical and experimental results, the consideration of a number of vibration modes equal to 10 seems more feasible, encompassing the range of modes and frequencies excited by the pedestrian's movement in the staircase response.

It is important to note that the use of Modal Superposition, including 10 vibration modes, should be employed in a more comprehensive number of real staircases to verify whether the considered number of modes, or an approximate value, consistently gives rise to precise response estimations. Nevertheless, it seems that the dynamic behaviour of structures subject to human induced vibrations will mostly governed by the low frequency modes, which is also in agreement with Davis (2008).

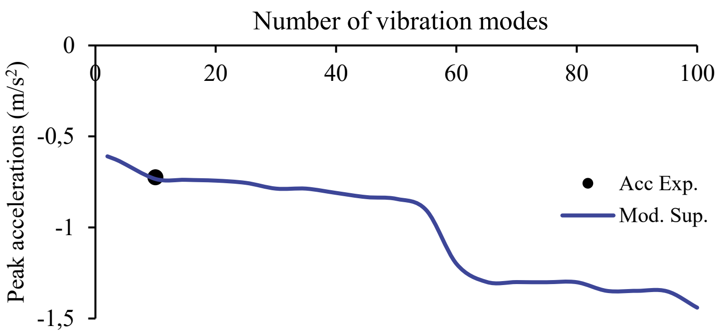


Fig. 5. Comparison between the numerical peak accelerations with decreasing number of vibration modes and the experimental peak acceleration.

5. Conclusions

The accelerations obtained by the Direct Integration, Modal Superposition and Duhamel’s integral in the analysed SDOF simply supported beam were identical, showing that in systems mostly governed by its fundamental frequency, all three methods are valid and will presumably yield to the same results.

When applying the Direct Integration and the Modal Superposition to the studied steel staircase, a more complex MDOF system, the results are no longer coincident. The accelerations obtained by the Direct Integration become considerably higher than those calculated by the Modal Superposition and experimentally measured. Conversely, the accelerations obtained with the Modal Superposition are very close to the experimental values.

Direct Integration led to an overestimated response, since the staircase FE model took into account the contribution of all vibration modes and an extensive number was not comparable with the real structure modes. The use of Modal Superposition allowed having control over the number of vibration modes to be considered, thus filtering the response contribution of modes with higher natural frequencies that were not of interest.

In the various analysis performed, it was verified that the numerical accelerations reach values closer to the experimental, when considering a number of vibration modes equal to 10, which demonstrates that only the low frequency modes were in the frequency range excitable by the human walking. However, this interval should be tested in a wider range of practical cases to assess if can be safely applied in the design of new staircases, without the loss of significant accelerations peaks. Necessarily, when employing the Modal Superposition method, the number of vibration modes should also be defined considering the staircase's stiffness, mass (important factor in impulsive responses), fundamental frequency and natural frequencies lower than 16 Hz, which may lead to a resonance build-up.

Based on the work developed, due to the increasing tendency for building flexible staircases, which are highly susceptible to human induced vibrations, and the still existing limitations for FE models successfully predict all vibrations modes and, consequently, the real structure's response, the Modal Superposition is currently the less time-consuming and more feasible method for design routine usage, in order to accurately estimate the staircase's dynamic behaviour.

Acknowledgements

This work was financially supported by: Base Funding - UIDB/04708/2020 of the CONSTRUCT - Instituto de I&D em Estruturas e Construções - funded by national funds through the FCT/MCTES (PIDDAC).

References

Andrade, P., 2017. Vibrações em Escadas Metálicas Induzidas por Peões, Civil Engineering and Geology. University of Madeira, MSc. thesis. Madeira, Portugal.

Andrade, P., Santos, J., Escórico, P., 2017a. Application of the Effective Impulse Approach to Stairs, 2nd International Conference on Structural Integrity, ICSI 2017, Funchal, Madeira, Portugal, pp. 1318-1325.

Andrade, P., Santos, J., Maia, L., 2017b. Reinforcement Measures to Reduce the Human Induced Vibrations on Stair Steps - A Case Study, 2nd International Conference on Structural Integrity, ICSI 2017, Funchal, Madeira, Portugal, pp. 1310-1317.

Barret, A.R., 2006. Dynamic Testing of In-Situ Composite Floors and Evaluation of Vibration Serviceability Using Element Method. Virginia Polytechnic Institue and State University, Ph.D. thesis. Virginia, USA.

Bishop, N.W.M., Willford, M., Pumphrey, R., 1995. Human induced loading of flexible staircases. Safety Science 18, 261-276

Davis, B., Avci, O., 2015. Simplified vibration serviceability evaluation of slender monumental stairs. Journal of Structural Engineering 141, 1-9.

Davis, B., Murray, T.M., 2009. Slender monumental stair vibration serviceability. Journal of Architectural Engineering 15, 111-121.

Davis, D.B., 2008. Finite element modeling for prediction of low frequency floors vibrations due to walking. Virginia Polytechnic Institute, Ph.D. thesis. Virginia, USA.

González, H., 2013. Numerical simulation of human induced vibrations of stairs. Bauhaus-Universität Weimar, MSc. thesis. Weimar, Germany.

Kasperski, M., Czwikla, B., 2012. A refined model for human induced loads on stairs, Proceedings of the 30th IMAC, A Conference on Structural Dynamics, Jacksonville, Florida, USA

Meinhardt, C., Zabel, V., Gonzalez, H., 2014. An in-situ case study induced vibrations on slender staircases, Proceedings of the 9th International Conference on Structural Dynamics, EURODYN 2014, Porto, Portugal

Santos, J., Andrade, P., Escórcio, P., 2019. Pre-design of laterally supported stair steps. Engineering Structures 182, 51-61.