Journal of Physics D: Applied Physics

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INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS D: APPLIED PHYSICS

J. Phys. D: Appl. Phys. **36** (2003) 603–614 PII: S0022-3727(03)54322-4

Heating of refractory cathodes by high-pressure arc plasmas: II

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Received 3 October 2002
Published 26 February 2003
Online at stacks.iop.org/JPhysD/36/603

Abstract
Solitary spots on infinite planar cathodes and diffuse and axially symmetric spot modes on finite cathodes of high-pressure arc discharges are studied in a wide range of arc currents. General features are analysed and extensive numerical results on planar and cylindrical tungsten cathodes of atmospheric-pressure argon arcs are given for currents of up to 100 kA. It is shown, in particular, that the temperature of cathode surface inside a solitary spot varies relatively weakly and may be estimated, to the accuracy of about 200–300 K, without actually solving the thermal conduction equation in the cathode body. Asymptotic behaviour of solutions for finite cathodes in the limiting case of high currents is found and confirmed by numerical results. A general pattern of current–voltage characteristics of various modes on finite cathodes suggested previously on the basis of bifurcation analysis is confirmed. A transition from the spot modes on a finite cathode in the limit of large cathode dimensions to the solitary spot mode on an infinite planar cathode is studied. It is found that the solitary spot mode represents a limiting form of the high-voltage spot mode on a finite cathode. A question of distinguishing between diffuse and spot modes on finite cathodes is considered.

1. Introduction

The present paper represents a second part of the work concerned with a detailed study of various aspects of the interaction of a high-pressure plasma with refractory cathodes. In the first part [1], a solution on the plasma side was considered, results of numerical modelling of the diffuse mode of current transfer under conditions of a low-current arc were given and a good agreement with the experimental data was shown. The aim of the present paper is to report on results of numerical modelling of diffuse and axially symmetric spot modes in a wide current range and to analyse, on the basis of numerical results and analytical treatment, general features of interaction of a high-pressure plasma with refractory cathodes in a wide current range.

A brief review of works concerned with modelling of multiple (spot and diffuse) modes of current transfer to arc cathodes has been given in [1]. Here, we mention only that a number of important questions still remain unresolved. For example, the pattern of current–voltage characteristics corresponding to different modes which has been sketched on the basis of qualitative analysis in [2] seems not to be confirmed by the numerical modelling [3, 4]. In particular, in [4], no bifurcation points have been detected; one of the spot modes just terminated (at one of the points termed critical) instead of turning back or joining another solution, which is not a behaviour typical of multiple solutions and does not fit in the pattern [2]. Other unclear questions are those of properties of the solitary spot mode, of a transition from this mode to a spot mode on a finite cathode, and of what are the most appropriate definitions of diffuse and spot modes. All these questions will be addressed in the present work.

2. Solitary spot

In this section, we consider an axially symmetric solitary spot on an infinite planar cathode. One can expect that this mode represents a limiting form of one of the spot modes on a finite cathode; this limiting transition will be studied in section 3.2.

2.1. The model

Introducing the heat flux potential,

$$\psi = \int_{T_c}^{T} \kappa(T)\,\mathrm{d}T, \qquad (1)$$

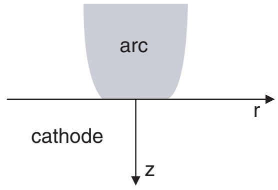

Figure 1. Schematic of the model of a solitary spot.

one can write the steady-state thermal conduction equation in the cathode body as the Laplace equation, which under the assumption of axial symmetry reads

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\psi}{\partial r}\right)+\frac{\partial^2\psi}{\partial z^2}=0. \qquad (2)$$

Here r, z are cylindrical coordinates with the origin at the centre of the spot and with the z-axis directed from the surface into the cathode body (see figure 1), T is the local cathode temperature, $\kappa=\kappa(T)$ is the thermal conductivity of the cathode material (a given function of T), and T_c is the temperature far from the spot (a given parameter).

Equation (2) should be solved in the half-space $z\geqslant 0$ (the cathode body) with the boundary conditions

$$\frac{\partial\psi}{\partial z}=-q(T_w,U) \qquad (3)$$

at the cathode surface, $z=0$, and

$$\psi\to 0 \qquad (4)$$

inside the cathode far from the spot, $\{r+z\to\infty, z\geqslant 0\}$. Here q is the density of the energy flux from the plasma to the cathode surface which is considered to be a known function of the local surface temperature T_w and of the near-cathode voltage drop U; the function $q(T_w,U)$ is calculated in this work by means of the model [1].

Function q must satisfy certain conditions for a solitary spot to exist. First, it must vanish at $T_w=T_c$: $q(T_c,U)=0$. This condition follows from the requirement that there is no energy flux far from a solitary spot or, alternatively, from the requirement that the problem (2)–(4) have, apart from a non-trivial solution describing the spot, also the trivial solution $T\equiv T_c$ describing a situation when there is no spot at the point considered. Furthermore, function q must vanish at $T_w\to T_c$ rapidly enough. Indeed, if the spot is solitary, the total power removed from the spot by thermal conduction is given by the improper integral

$$Q_c=2\pi\int_0^\infty q[T_w(r),U]r\,\mathrm{d}r. \qquad (5)$$

At large distances from a solitary spot, the difference (T_w-T_c) tends to zero proportionally to $1/r$. Assuming that $q\approx C_1(T_w-T_c)^n$ at $T_w\to T_c$ (here C_1 and $n>0$ are quantities given), one finds that $q[T_w(r),U]$ tends to zero proportionally to $1/r^n$ and the integral (5) converges provided that $n>2$. The physical sense of this condition is quite clear: the energy flux density must vanish rapidly enough far from the spot, otherwise a perturbation created by the spot would not be localized.

The above conditions are not formally satisfied for the function q described in [1] due to the radiation losses (unless $T_c=0$). Therefore, in calculations of a solitary spot the radiation losses in the model [1] of function q were set equal to zero for $T_w<T_{co}$, where T_{co} is a certain temperature higher than T_c.

The computation time can be drastically reduced by transforming the (two-dimensional) non-linear boundary-value problem (2)–(4) to a one-dimensional integral equation for the surface temperature, as it was done in [5, 6]. However, in this work such a transformation is not employed and the two-dimensional problem is solved by means of the iterative approach [1].

A boundary which separates the calculation domain from the rest of the cathode should be chosen far enough from the origin for the solution to be independent of the exact position of the boundary. A straightforward way to formulate a boundary condition at this boundary is to shift equation (4) from infinity to the boundary, i.e. to set $\psi=0$. It is advisable, however, to use another boundary condition which would allow one to exclude from calculations a region very far from the spot where a solution has already reached its asymptotic behaviour although still is non-zero. Such a boundary condition can be derived as follows. Asymptotic behaviour of a solution at large distances from the spot reads to a first approximation

$$\psi=\frac{C_2}{\rho}, \qquad (6)$$

where $\rho=\sqrt{r^2+z^2}$ is the distance from the centre of the spot and C_2 is a constant. Differentiating the right-hand side, one arrives at the desired asymptotic boundary condition:

$$r\frac{\partial\psi}{\partial r}+z\frac{\partial\psi}{\partial z}+\psi=0. \qquad (7)$$

In the present calculations, the boundary condition (7) was used. Contributions of the part of the cathode surface outside the calculation domain to the power removed by thermal conduction, to the irradiated power, and to the integral electric current I collected by the cathode have been neglected.

2.2. General: the upper limit of the cathode temperature and the lower limit of the temperature inside low-voltage solitary spots

A number of qualitative considerations on a spot on a semi-infinite cathode was given in [7, 8]. In this section, these considerations are gathered together and analysed with the aim to present as complete and general picture as possible.

The dependence of q on T_w at fixed U calculated by means of the model [1] for two values of the voltage drop is shown in figure 2. (All numerical calculations of this work refer to the atmospheric-pressure argon plasma and a tungsten cathode unless otherwise is indicated; other parameters are the same as in [1].) Main features of this dependence are as follows. q is negative at low T_w, when radiation cooling of the cathode surface prevails over its heating by the plasma. At higher T_w, q turns positive, then passes through a maximum (at low U) or through a maximum, then through a minimum and then through another maximum (at high U). At still higher T_w, thermionic cooling exceeds the combined heating by the ions and plasma electrons and q again turns negative. We shall designate the temperature values at which q changes sign by T_1 and T_2, being $T_1 < T_2$. In the case of high voltages, when the dependence of q on T_w has two maxima, the temperature values corresponding to the maxima will be designated $T_1^{(\max)}$ and $T_2^{(\max)}$ and the temperature value corresponding to the minimum will be designated $T^{(\min)}$, being $T_1 < T_1^{(\max)} < T^{(\min)} < T_2^{(\max)} < T_2$. In the case of low voltages, when the dependence of q on T_w has only one maximum, the respective temperature will be designated $T_2^{(\max)}$.

Obviously, all the above-introduced temperature values depend on U. However, this dependence is not very strong, at least in the case of the temperatures $T_2^{(\max)}$ and T_2, which are the most relevant for the subsequent analysis. An example of functions $T_1^{(\max)}(U)$, $T_2^{(\max)}(U)$, and $T_2(U)$ can be found in figure 3. One can see that T_2 and $T_2^{(\max)}$ indeed do not change much with U. Note that there is a narrow range of U close to 25 V in which the dependence of q on T_w has a plateau; this range is intermediate between low U at which the dependence has one maximum and high U at which the dependence has two maxima. One virtually cannot speak of maxima in this range. Therefore, quantities $T_1^{(\max)}(U)$ and $T_2^{(\max)}(U)$ in figure 3 are not shown in this range.

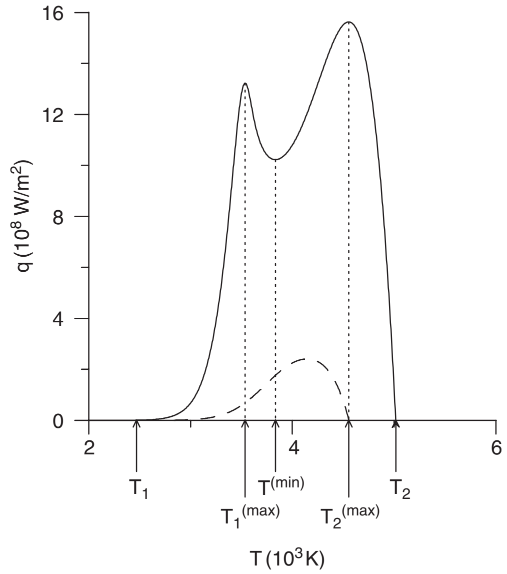

Figure 2. Dependence of q on T_w for the atmospheric-pressure argon plasma and a tungsten cathode. ——: $U = 50$ V. - - - -: $U = 15$ V.

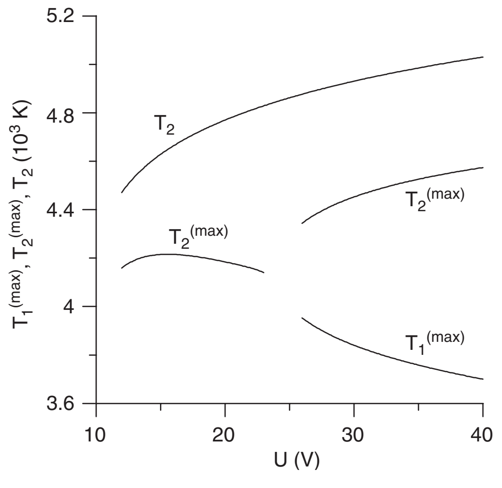

Figure 3. Characteristic points of the dependence of q on T_w. Atmospheric-pressure argon plasma and tungsten cathode.

In [7], a number of qualitative considerations was given on a spot on a semi-infinite cathode in the case when the dependence of q on T_w has one maximum. It was indicated that, as it appears, a stable situation must be reached with a spot temperature somewhere between that at which this dependence attains the maximum value and that at which q becomes negative (or, in terms of the present work, somewhere between $T_2^{(\max)}$ and T_2). This assumption has been confirmed by results of numerical calculations [6], which have shown that the temperature inside the spot on a semi-infinite cathode in the case when the dependence of q on T_w has one maximum varies from $T_2^{(\max)}$ at the spot edge to a somewhat higher value at the centre of the spot, the latter value being below T_2.

Thus, one can obtain an estimate of the temperature inside a spot on a large cathode at low voltages by means of finding and analysing a solution on the plasma side: it is sufficient to calculate the function $q(T_w, U)$ and to find values $T_2^{(\max)} = T_2^{(\max)}(U)$ and $T_2 = T_2(U)$ at which q as a function of T_w for a given U attains the maximum value and turns negative, respectively. It should be emphasized that there is no need to solve the thermal conduction equation in the cathode body for such an estimate.

The conclusion that the temperature of a spot on a semi-infinite cathode, in the case of a dependence of q on T_w with one maximum, varies between $T_2^{(\max)}$ and T_2, obviously has a limited applicability. In particular, it applies only to 'well-developed' spots in the sense that, for example, the hottest point of a finite cathode operating in the diffuse mode may easily have a temperature below $T_2^{(\max)}$; examples can be found in [1]. On the other hand, the upper limit of the cathode surface temperature established by the above statement is quite general: one can affirm that the temperature of the hottest point of a cathode without significant Joule heating is always below the temperature T_2.

The latter proposal can be proved as follows [8]. The heat flux potential, being a solution of the Laplace equation, is a harmonic function of spatial coordinates. Consider a point of the cathode at which this function attains its maximum value. According to the maximum principle for harmonic functions (e.g. [9]), this point is positioned on the cathode surface rather than inside the cathode. Since the point under consideration is a point of maximum of the function ψ, the derivative $\partial\psi/\partial n$ cannot be negative at this point (here n is a direction locally orthogonal to the cathode surface and directed outside the cathode). Since $\partial\psi/\partial n = q$ at the cathode surface, the function q is non-negative at this point, which means that the local value of the function ψ does not exceed ψ_2 (the value given by equation (1) for $T = T_2$). Thus, the maximum value of the function ψ cannot exceed ψ_2. It follows that the temperature does not exceed T_2 at any point of the cathode.

A physical sense of this conclusion is quite clear: values of the temperature exceeding T_2 can be maintained neither by the energy flux from the plasma (which is negative for $T > T_2$), nor by the heat flux from adjacent points of the cathode body (this flux could be originated only by points of the surface where the energy flux from the plasma is positive, however heat cannot propagate from these points to a hotter point).

It should be emphasized that the conclusion that the temperature of the cathode cannot exceed T_2 applies to any model of the plasma–cathode interaction in which the energy flux to the cathode surface is governed by the local surface temperature and by the near-cathode voltage drop which is constant along the current-collecting surface, and in which Joule heating inside the cathode body is neglected. In particular, this conclusion applies regardless of the number of maxima of the dependence of q on T_w at fixed U. This conclusion applies also to cathodes of any shape operating in any mode of current transfer.

This conclusion allows one to rapidly estimate the upper limit of the temperature of the hottest point of a thermionic cathode. For example, using the data on T_2 shown in figure 3, one can say that the temperature of the hottest point of a tungsten cathode in atmospheric-pressure argon plasma at any arc current in any mode of current transfer does not exceed approximately 4500–5000 K.

2.3. Results of numerical simulation

Range of values of the near-cathode voltage drop investigated in this work is up to 40 V. Calculations of this section have been performed with $T_c = 293$ K and $T_{co} = 500$ K in the range of U down to 11.8 V, which corresponds to the current range of up to approximately 100 kA. The calculation domain was $\{r, z \leqslant 5\,\text{mm}\}$ in the range $40\,\text{V} \geqslant U \geqslant 30\,\text{V}$, $\{r, z \leqslant 10\,\text{mm}\}$ in the range $30\,\text{V} \geqslant U \geqslant 20\,\text{V}$, $\{r, z \leqslant 20\,\text{mm}\}$ in the range $20\,\text{V} \geqslant U \geqslant 15\,\text{V}$, $\{r, z \leqslant 40\,\text{mm}\}$ in the range $15\,\text{V} \geqslant U \geqslant 13\,\text{V}$ and $\{r, z \leqslant 80\,\text{mm}\}$ in the range $13\,\text{V} \geqslant U \geqslant 11.8\,\text{V}$. A non-uniform grid was used.

In figure 4, the current–voltage characteristic of a solitary spot is shown. One can see that this characteristic is falling: the near-cathode voltage drop is quite high at low currents and slowly decreases with an increase of current. In order to verify reliability of the results, two variations have been made (in turn): dimensions of the calculation domain have been duplicated; T_{co} was set to 1000 K. In both cases, the current–voltage characteristic has not changed, to the graphic accuracy.

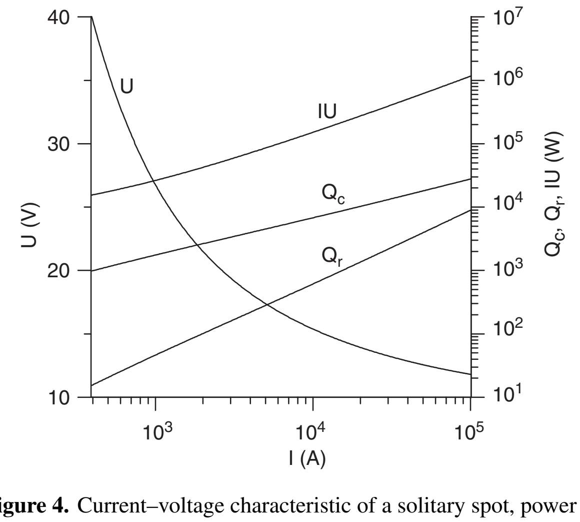

Figure 4. Current–voltage characteristic of a solitary spot, power removed by thermal conduction, irradiated power, and power input into the near-cathode layer.

Also shown in figure 4 is power Q_c removed by thermal conduction, the irradiated power Q_r, and power input into the near-cathode layer, IU. One can see that the total power transported to the cathode, $Q_c + Q_r$, is much smaller than the power input into the near-cathode layer, which means that most of the power deposited in the near-cathode layer is transported by electrons to the bulk plasma. This is in contrast with calculations of the diffuse mode at low currents reported in [1], a reason being that the electron temperature in the near-cathode layer in the diffuse mode at low currents is considerably smaller than that in the spot mode. This means, in particular, that the determination of the near-cathode voltage drop by means of calorimetric measurements can hardly be reliable in the solitary spot mode.

One can see from figure 4 also that the irradiated power is much smaller than the power removed by thermal conduction. Again, this is in contrast with calculations [1], a reason being that cathodes considered in [1] were thin.

Distributions along the cathode surface of the surface temperature and of the densities of the energy flux and electric current from the plasma to the surface for several values of the arc current are shown in figure 5. While distributions of the surface temperature $T_w(r)$ and of the current density $j(r)$ are monotonic, the distribution of the energy flux density $q(r)$ is non-monotonic and has two maxima (at $U = 40$ V and 30 V) or one maximum (in all the other cases). Since the function $T_w(r)$ is monotonic, the non-monotony of the function $q(r)$ is obviously a consequence of the non-monotony of the dependence of q on T_w at fixed U.

The distributions shown in figure 5 have the same features that were observed in the numerical modelling [6] and were used for constructing an approximate asymptotic theory of spots in [6, 10, 11]: the surface temperature inside the spot changes relatively weakly and varies from $T_2^{(\max)}$ to a value which is somewhat lower than T_2; the energy flux density vanishes rather abruptly outside the spot, i.e. the spot has a more or less distinct edge. At low voltages (high currents) when the dependence of q on T_w at fixed U has only one falling section $[T_2^{(\max)}, T_2]$, this similarity should have been expected. As far as high voltages (low currents) are concerned, this result means that the presence of another falling section, $[T_1^{(\max)}, T^{(\min)}]$, does not change the structure of a solution. Indeed, the surface temperature within the spot corresponds to the falling section $[T_2^{(\max)}, T_2]$ also at high voltages; the effect produced on the distributions of surface temperature in figure 5(a) by the presence of another falling section is not apparent, although it is manifestant through the current density distribution (the function $j(r)$ decreases slower in the respective region). A variation of the current density inside the spot is not very considerable, although more pronounced than that of the temperature.

It can also be seen from figure 5 that the maximum temperature inside the spot does not increase with an increase of current; in fact, it weakly decreases. This feature is a consequence of the fact that the temperature T_2 is a slowly increasing function of the voltage drop U (see figure 3) while the current–voltage characteristic of a solitary spot $U(I)$ is a slowly decreasing function. This feature can be observed also in the current density distribution, although the variation of the maximum current density with the discharge current is more

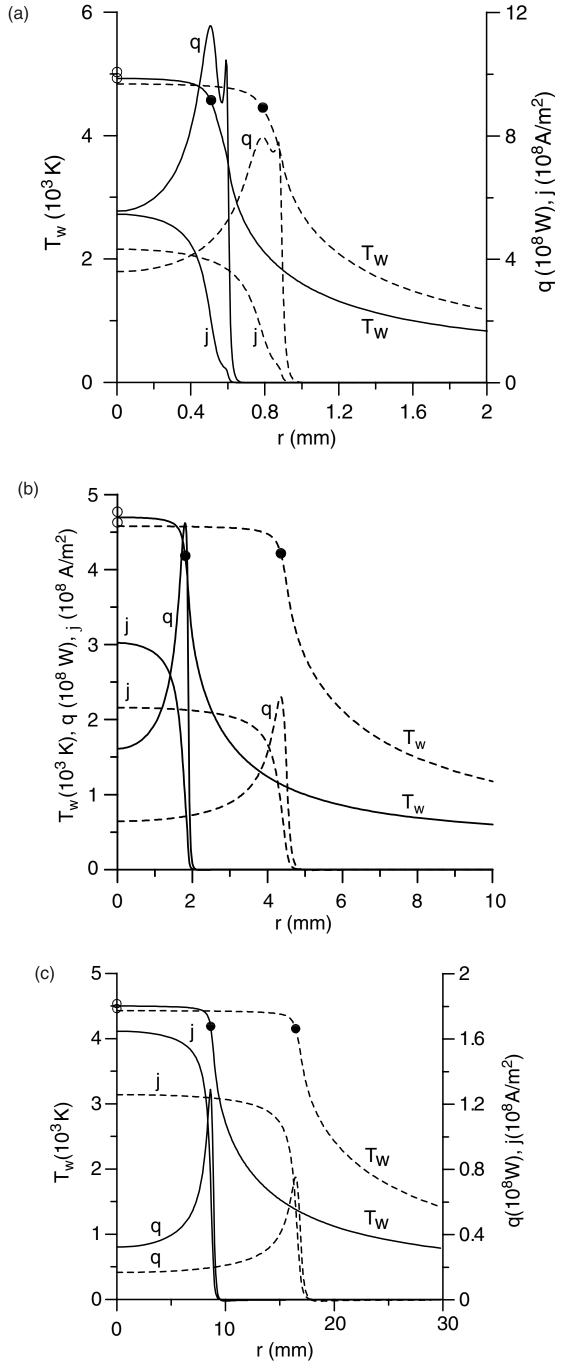

Figure 5. Distribution of the cathode surface temperature, the energy flux density, and of the electric current density in a solitary spot. ●: points at which $T_w = T_2^{(\max)}$. ○: T_2. (*a*) ——: $I = 394$ A ($U = 40$ V). - - - -: 743 A (30 V). (*b*) ——: 2.65 kA (20 V). - - - -: 11.7 kA (15 V). (*c*) ——: 35.8 kA (13 V). - - - -: 101 kA (11.8 V).

pronounced than that of the temperature. One can say that the spot intensity does not change much with an increase of current and the current increase is ensured by an increase of the spot area. This feature has been noticed in [8] and is similar, in a sense, to the effect of normal current density observed on cold glow cathodes (as the discharge current increases, the normal spot expands, local parameters in the spot being virtually constant).

One can see from figure 3 that the temperature range $[T_2^{(\max)}, T_2]$ is relatively narrow. This explains the above-mentioned weak variation of the temperature (and of other parameters) inside the spot. Furthermore, one can estimate the temperature inside the spot as $(T_2^{(\max)}+T_2)/2$ to the accuracy of about 200–300 K. The spot temperature estimated in this way is governed mainly by the work function of the cathode material; dependence on other parameters is relatively weak. The latter conclusion conforms to the fact revealed by experiments with atmospheric-pressure argon arcs with cathodes made of thoriated or pure tungsten with a current of the order of a few hundred amps [12–16]: the spot temperature is virtually independent of the exact value of the discharge current and of the cathode geometry, being about 3500 K in the case of a thoriated-tungsten cathode and about thousand degrees higher in the case of a cathode made of pure tungsten (for which the work function is considerably higher than that for thoriated tungsten).

One can see from figure 5 that the temperature inside the spot exceeds the melting temperature of tungsten, which is approximately 3700 K. Hence, there is a melt pool inside the spot and the shape of the surface may be distorted. However, since the present work is aimed mainly at establishing qualitative results, this phenomenon is left beyond its scope. Note that such an approach is similar to the one employed in the modelling of plasma–cathode interaction in arc welding (e.g. [17]), in which melting of cathodes is not usually taken into account and the cathode shape is assumed to be fixed in space and time.

3. Modes on a finite cathode

In this section, an asymptotic solution for a finite cathode in the limiting case of high currents is presented first. After that, simulation results on the diffuse and axially symmetric spot modes on cylindrical cathodes in a wide current range are given.

3.1. General: asymptotic solution in the limiting case of high currents

One can expect that every point of the current-collecting surface of the cathode attains at currents high enough the above-mentioned value T_2, at which the dependence of q on T_w at U given turns negative. Then the limiting form of the current–voltage characteristic at high currents is given by the formula

$$I = \int j[T_2(U), U]\,\mathrm{d}S, \tag{8}$$

where $j = j(T_w, U)$ is a function describing dependence of the current density from the plasma to the cathode surface on the local surface temperature and the near-cathode voltage drop [1] and the integral is extended over the whole current-collecting surface of the cathode. After the current–voltage characteristic $U = U(I)$ described by this formula has been calculated, one will be able to determine the temperature of the current-collecting surface of the cathode in terms of the arc current:

$$T_w = T_2[U(I)]. \tag{9}$$

Equations (8) and (9) represent a complete asymptotic solution of the problem in the limiting case of high currents. It should be emphasized that this solution can be obtained without actually solving the thermal conduction equation in the cathode body.

One can conclude that if more than one mode exists in the range of high currents, all these modes are asymptotically similar, i.e. tend to the same limiting form described by equations (8) and (9).

3.2. Results of numerical simulation

We consider cathodes in the form of a right circular cylinder and introduce cylindrical coordinates r, z with the origin at the centre of the front surface of the cathode and with the z-axis directed from the front surface into the cathode body. Equation (2) is solved in the cathode body (domain $\{r \leqslant R, 0 \leqslant z \leqslant h\}$, where R is the radius of the cathode and h is its height). The boundary condition at the front surface of the cathode, $\{r \leqslant R, z = 0\}$, is given by equation (3). The boundary condition at the lateral surface, $\{r = R, 0 \leqslant z \leqslant h\}$, is given by equation (3) with z replaced by $-r$. The bottom of the cathode, $\{r \leqslant R, z = h\}$, is externally cooled and the respective boundary condition is

$$T = T_c. \tag{10}$$

In calculations, the cooling temperature T_c was set equal to 293 K as in the preceding section.

In figure 6, current–voltage characteristic of various modes of current transfer to a cathode are shown. One can see that the current–voltage characteristic of the diffuse mode has two sections, one falling (*AB*) and another rising (*BC*), separated by the point of minimum B. The point H designates a current value starting from which the temperature of the hottest point of the cathode (which is the edge of the front surface) exceeds $T_2^{(\max)}$. One can see that this point

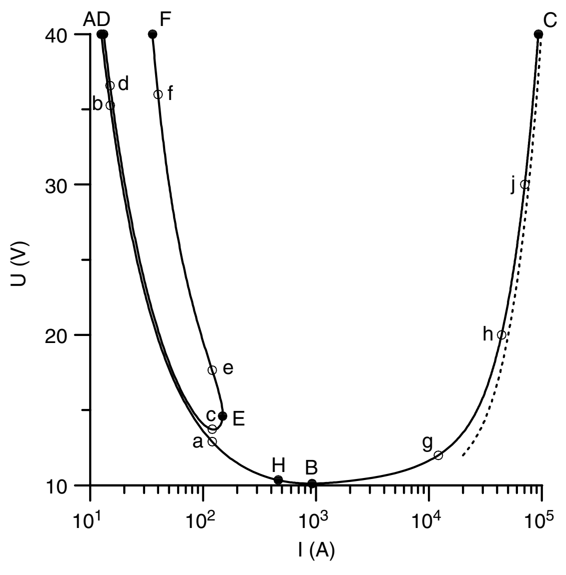

Figure 6. Current–voltage characteristics of various modes of current transfer to a cathode. $R = 2$ mm, $h = 10$ mm. *ABC*: the diffuse mode. *DEF*: the spot mode. $\cdots\cdots$: asymptotic behaviour at high currents.

is positioned not far from the point of minimum. Thus, one can say roughly that the falling section of current–voltage characteristic is associated to the rising section of the dependence of q on T_w and *vice versa*. The current–voltage characteristic of the spot mode has two branches *ED* and *EF* separated by a turning point E. Both branches are falling, except a small section of the branch *ED*. Branch *EF*, which is characterized by higher values of the near-cathode voltage than the branch *ED*, will be referred to in the following as the high-voltage branch, while branch *ED* will be termed the low-voltage branch. The most part of the low-voltage branch is very close to the current–voltage characteristic of the diffuse mode.

The above properties conform to qualitative considerations [2]; cf in particular, figure 5 of [2] where the lines *AEF* and $D'DD''$ represent the current–voltage characteristics of the diffuse mode and of the axially symmetric spot mode, respectively. Note that the diffuse and spot-mode solutions do not join under conditions of the present modelling, which is not surprising since both solutions are two-dimensional; a few further comments on this point will follow.

Thus, the general pattern of current–voltage characteristic of various modes found in [2] on the basis of qualitative considerations has been confirmed by the numerics. On the other hand, one should suspect that the spot solutions found in [3,4], which just terminate instead of turning back or joining another solution, have been found only in a part of their existence regions.

One can see from figure 6 that the diffuse mode is the only one possible at $I \gtrsim 150$ A. As it should have been expected, its current–voltage characteristic tends in the limit of high currents to the asymptotic form described by equation (8) (with the integral extended over the front and lateral surfaces of the cathode), which is shown by the dotted line.

It should be emphasized that the present modelling is aimed at finding all steady-state axially symmetric solutions without regard of their stability. Note that while in the modelling the diffuse mode exists in the whole range of currents considered, in the experiment the diffuse mode ceases to be observed at low currents, which indicates that this mode at low currents is unstable. The question of calculation of the limit of stability will be treated in the third part of this work.

Temperatures at the centre and at the edge of the front surface of the cathode for the diffuse and spot modes as functions of current are shown in figure 7. In the diffuse mode in the current range $I \lesssim 3$ kA, the temperature at the centre is lower than that at the edge. This effect has been observed in the modelling [1], however under the present conditions it is more pronounced: the temperature at the edge can exceed the temperature at the centre by more than 500 K, which is due to a higher value of the ratio R/h. As it should have been expected, the temperatures at the centre of the cathode and at the edge of the front surface become at high currents equal between themselves and to the limiting temperature described by equations (8) and (9), which is shown by the dotted line.

In the spot mode, the temperature at the centre is in all the cases higher than that at the edge. The temperature difference on the high-voltage branch *EF* is considerably higher than that on the low-voltage branch *ED*. On the high-voltage branch, the temperature at the centre decreases with an increase of

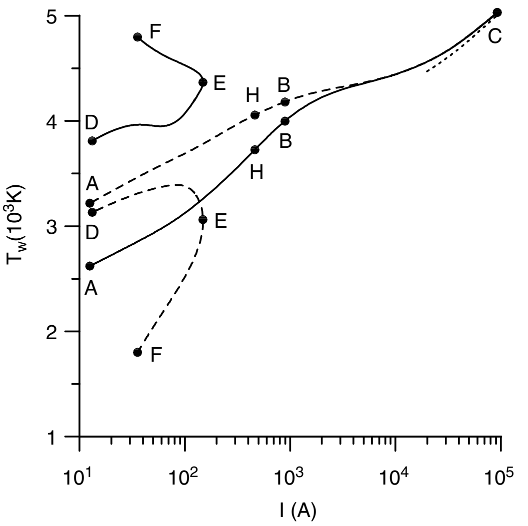

Figure 7. Temperature at the centre and at the edge of the front surface of the cathode for different modes of current transfer. $R = 2$ mm, $h = 10$ mm. ——: temperature at the centre. - - - -: temperature at the edge. ······: asymptotic behaviour at high currents.

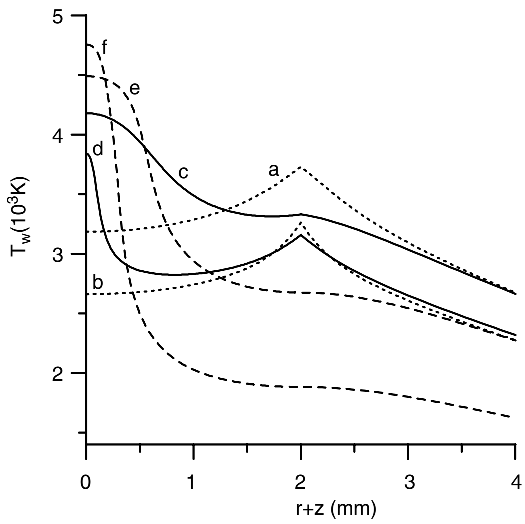

Figure 8. Distributions of the temperature along the cathode surface for states referring to various modes of current transfer. $R = 2$ mm, $h = 10$ mm.

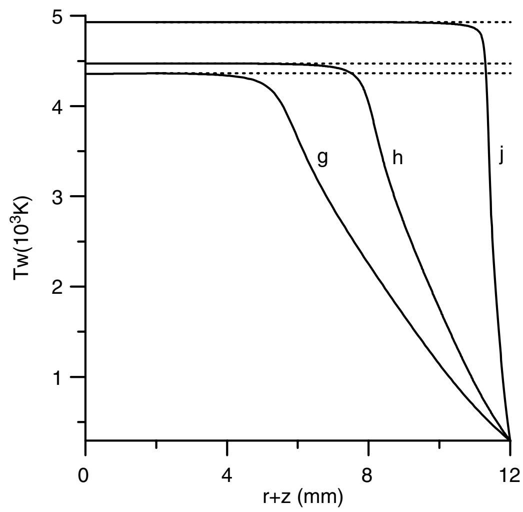

Figure 9. Temperature distributions along the cathode surface for states referring to the diffuse mode at high currents. $R = 2$ mm, $h = 10$ mm. ······: limiting values of the cathode temperature.

current, while the temperature at the edge increases. On the most part of the low-voltage branch, both the temperature at the centre and at the edge increase with an increase of current, the temperature difference being approximately constant.

The temperature at the centre of the front surface of the cathode in the spot mode exceeds the melting temperature of tungsten, while the temperature at the edge is below the melting temperature. (Note that the former is not necessarily the case in other conditions; it is known from the experiment that the spot temperature also may be below the melting temperature.) In the diffuse mode at high currents, temperatures both at the edge and at the centre exceed the melting temperature and the tip of the cathode is melted and can change its form. Again, this phenomenon is left beyond the scope of the present work. Note that in the modelling of plasma–cathode interaction in arc welding (e.g. [17]) melting of cathodes is not usually taken into account and the cathode shape is assumed to be fixed in space and time both in the case of gas metal arc welding, when the role of cathode is played by a (large) workpiece, and in the case of gas tungsten arc welding, when the role of cathode is played by a (thin) auxiliary electrode.

Distributions of the temperature along the cathode surface are shown in figure 8 for several states which are indicated in figure 6 and refer to various modes of current transfer. (Note that the range $0 \leqslant r + z \leqslant 2$ mm in this figure and in figures 9 and 10 corresponds to the front surface while the range $r + z \geqslant 2$ mm corresponds to the lateral surface.) Both in the diffuse mode (states a and b) and on the low-voltage branch of the spot mode (states c and d), the temperature decreases with a decrease of current, a difference being that the hottest point is the cathode edge in the diffuse mode and the centre of the front surface on the low-voltage branch of the spot mode. Distributions on the high-voltage branch of the spot mode (states e and f) reveal a formation of a spot at the centre of the cathode. The spot becomes more pronounced as the current decreases: the temperature increases inside the spot and decreases outside, the spot radius decreases.

The limiting transition to the high-current asymptotic regime described by equations (8) and (9) is illustrated by figure 9, where distributions of the temperature of the cathode surface in the diffuse mode are shown for high values of the arc current. (Again, the respective states are indicated in figure 6.) The dotted lines represent the respective values of T_2. One can see that the temperature becomes at high currents constant and equal to T_2 at all points of the cathode surface except a small section of the lateral surface adjacent to the bottom, where T_w rapidly falls down to the cooling temperature.

In figure 10, the distribution of the energy flux density over the cathode surface is shown for several states belonging to the diffuse mode. In all the states, the distribution is non-monotonic. In the state j, the distribution has two maxima, which originates in the above-mentioned fact that the dependence of q on T_w at high U has two maxima. The distribution corresponding to the state b has only one

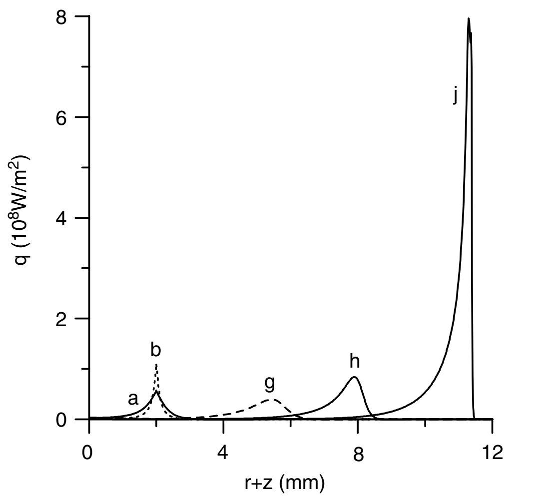

Figure 10. Distributions of the energy flux density over the cathode surface for states referring to the diffuse mode.

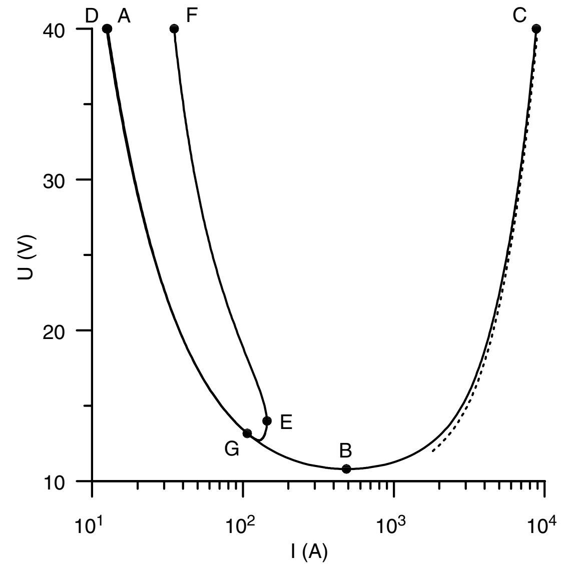

Figure 11. Current–voltage characteristics of various modes of current transfer to a cathode with an insulating lateral surface. $R = 2$ mm, $h = 10$ mm. *ABC*: the diffuse mode. *DEF*: the spot mode. $\cdots\cdots$: asymptotic behaviour at high currents.

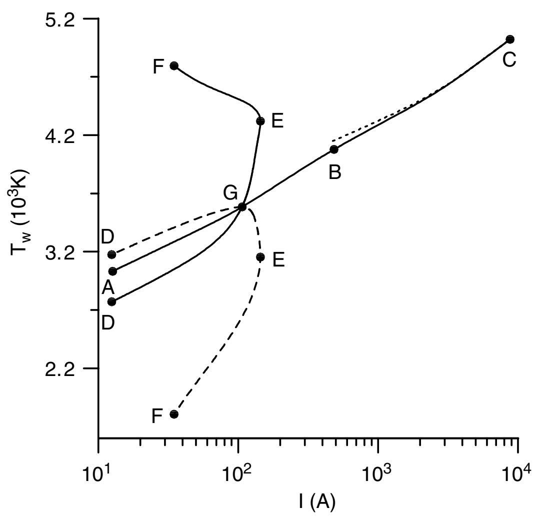

Figure 12. Temperature at the centre and at the edge of the front surface of the cathode with an insulating lateral surface for different modes of current transfer. $R = 2$ mm, $h = 10$ mm. ——: temperature at the centre. - - - -: temperature at the edge. $\cdots\cdots$: asymptotic behaviour at high currents.

maximum, although the respective voltage is higher than that in the state j. The reason is that the maximum temperature of the cathode is below $T_1^{(\max)}$ in the state b. At low currents, the maximum of the energy flux density occurs at the edge of the front surface. With an increase of current, the maximum moves onto the lateral surface and then in the direction to the bottom. At high currents, the energy flux density is very small at all points of the cathode surface except a small section adjacent to the bottom, where the temperature is different from T_2.

Calculations have been performed also for a cathode with an electrically and thermally insulating lateral surface. (Densities of the energy flux and of the electric current to the lateral surface were set equal to zero in these calculations.) For this model, a solution describing the diffuse mode is one-dimensional, $\psi = \psi(z)$, and it has been shown by means of the bifurcation analysis [2] that the diffuse-mode and spot-mode solutions join or, in other terms, that the spot-mode solutions branch off from the diffuse-mode solution.

Calculation results are shown in figures 11–13. Figure 11 is quite similar to figure 6. It is interesting to note, however, that most of the section *GD* of the low-voltage branch of the current–voltage characteristic of the spot mode in figure 11 is slightly below the current–voltage characteristic of the diffuse mode, while the low-voltage branch in figure 6 is always above the current–voltage characteristic of the diffuse mode. There is only one line referring to the diffuse mode in figure 12 since the temperature in the diffuse mode is the same at all points of the front surface of a cathode with an insulating lateral surface. The lines referring to the high-voltage branch in figure 12 are similar to those in figure 7 (in fact, the respective lines from the two figures are quite close if shown on the same scale). However, the two figures are different as far as the low-voltage branch is concerned: the temperature at the centre is lower than that at the edge on the section *GD* in figure 12 while in figure 7 the temperature at the centre is in the spot mode always higher than that at the edge. The reason of this difference is that the ring spot mode [2], which corresponds to the section *GD* in figure 12, does not occur under conditions of figure 7.

Also shown in figures 11 and 12 is asymptotic behaviour in the limiting case of high currents. (The integral in equation (8) was extended over the front surface of the cathode while calculating this behaviour.) Again, there is an agreement between the numerical and asymptotic results.

Point *G* in figures 11–13 has been determined by means of formulae [2] as a bifurcation point at which the first axially symmetric solution branches off. The present modelling shows that the spot- and diffuse-mode solutions indeed become exactly identical at this point. In figure 13, the asymptotic behaviour in the vicinity of the bifurcation point is shown of the temperatures in the spot mode at the centre of the front surface and at the edge, as determined by formulae [2]. One can see that the modelling results conform to the results of the bifurcation theory [2].

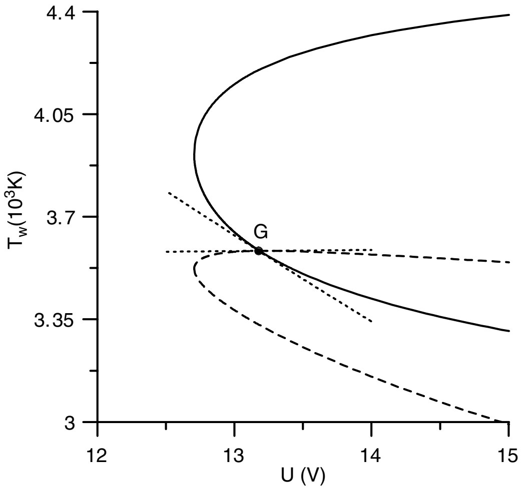

Figure 13. Temperature at the centre and at the edge of the front surface of the cathode with an insulating lateral surface in the spot mode in the vicinity of the bifurcation point. $R = 2$ mm, $h = 10$ mm. ——: temperature at the centre. - - - -: temperature at the edge. ······: asymptotic behaviour in the vicinity of the bifurcation point.

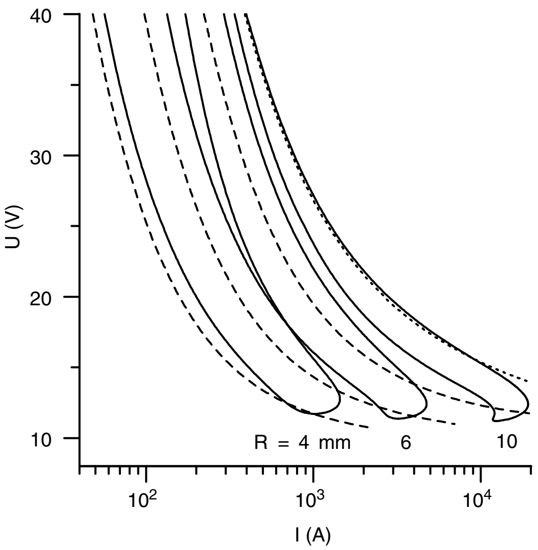

Figure 14. Current–voltage characteristics of various modes of current transfer to cathodes of different radii. $h = 10$ mm. ——: spot mode. - - - -: diffuse mode. ······: solitary spot.

In the rest of this section, an effect of variation of cathode dimensions is studied. One can see from figure 14 that an increase of the radius of the cathode at fixed cathode height causes a displacement of the current–voltage characteristics in the direction of higher currents. The point of minimum of the current–voltage characteristics of the spot mode is displaced into the direction of higher currents at a slightly decreasing voltage. The high-voltage branch of the current–voltage characteristic of the spot mode approximates the current–voltage characteristic of the solitary spot.

Distributions of the temperature along the surface of cathodes of different radii for various modes at the same current of 1 kA are shown in figure 15. (Note that the dotted lines represent the temperature distribution in a solitary spot with the current of 1 kA, while the circles represent values of the limiting cathode temperature T_2 evaluated at U which corresponds to the above-mentioned current for R in question.) In the diffuse mode (figure 15(a)), the temperature at the edge is little affected by a variation of R, while the temperature

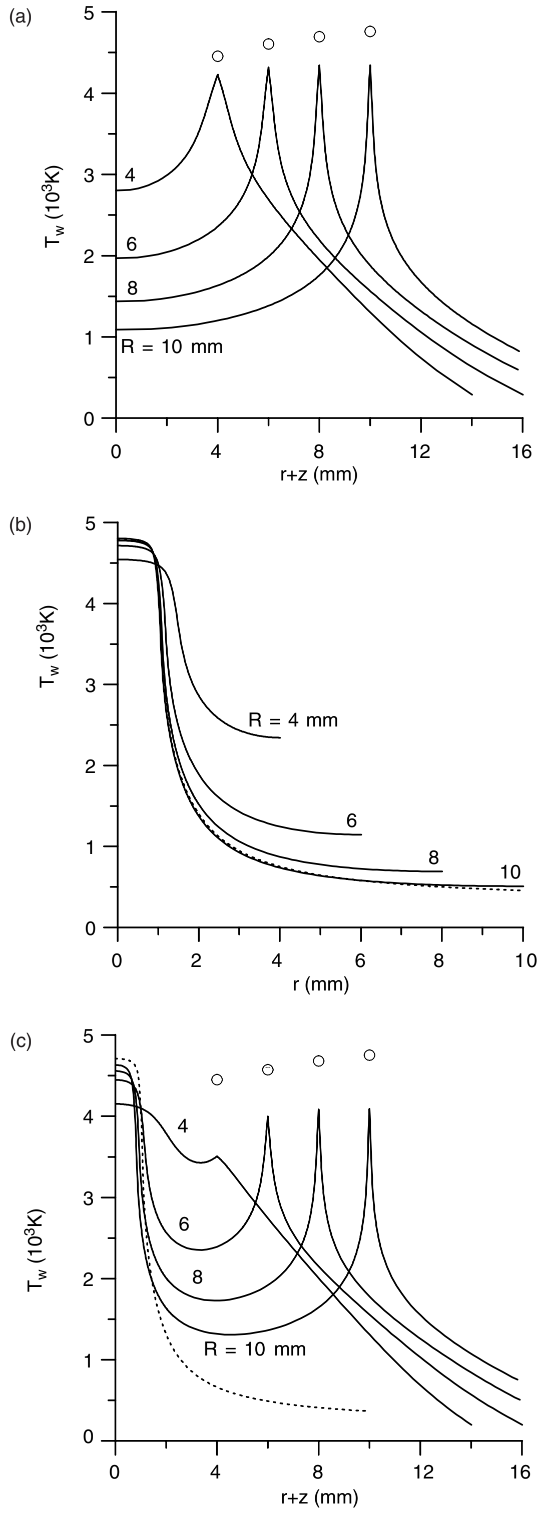

Figure 15. Distributions of the temperature along the surface of cathodes of different radii. $h = 10$ mm, $I = 1$ kA. ······: temperature distribution in a solitary spot. ○: limiting values of the cathode temperature. (a) Diffuse mode. (b) High-voltage branch of the spot mode. (c) Low-voltage branch of the spot mode.

at the centre of the cathode decreases considerably with an increase of R. One can speak in this case of a ring current structure attached to the cathode edge. As one can see from figure 15(a), values of the temperature at the spot edge are slightly lower than the limiting temperature, and the difference slowly increases with the cathode radius. A reason for the latter is that the bigger the cathode is, the more distant from the limiting high-current regime is a state corresponding to a fixed current in the diffuse mode.

The temperature distribution on the high-voltage branch of the spot mode (figure 15(b)) tends with increasing R to that in a solitary spot with the same current. This conforms to the above-mentioned fact that the high-voltage branch of the current–voltage characteristic becomes close to the current–voltage characteristic of the solitary spot. The temperature distribution on the low-voltage branch (figure 15(c)) at high R resembles a superposition of those corresponding to the diffuse mode and to the high-voltage branch: as R increases, a spot is being formed at the centre of the cathode while a ring structure is being formed at the edge.

As the cathode height increases at fixed radius (figure 16), the current–voltage characteristic of the diffuse mode is gradually displaced in the direction of lower currents (or lower voltages). The high- and low-voltage branches of the current–voltage characteristic of the spot mode become close and the characteristic on the whole is rapidly displaced in the direction of lower currents and higher voltages.

Calculations shown in figure 16 have been performed for $R = 0.75$ mm and the argon pressure of 2.6 bar with the aim to simulate conditions of the works [18–20], in which experimental data on the spot mode are given for tungsten cathodes of radii between 0.3 and 1 mm and of heights equal to or greater than 20 mm in the argon plasma under the pressure of 2.6 bar. One can suppose, on the basis of figure 16, that axially symmetric spot-mode solutions can hardly exist under these experimental conditions. It is interesting to note, however, that both branches of the current–voltage characteristic of the spot mode in figure 16 become at high h very close to the current–voltage characteristic of the diffuse mode, which conforms to the above-cited experiments.

Figures 14 and 15(b) give one an idea of what the relationship is between spot modes on large cathodes and the solitary spot mode on an infinite planar cathode: one can suppose that the solitary spot mode represents a limiting form of the high-voltage spot mode. In order to check this hypothesis, the current–voltage characteristic of the high-voltage mode on cathodes of equal and increasing radius and height has been calculated (figure 17). One can see that the current–voltage characteristic of the high-voltage mode indeed tends to the current–voltage characteristic of the solitary spot as the cathode dimensions increase. With an increase of the cathode dimensions, the voltage drop decreases: an increase of the cathode dimensions results in a better thermal insulation of the spot (i.e. in an increased distance between the spot and the cooled surface) and in a lower power necessary to maintain the discharge.

3.3. Definition of diffuse and spot modes

It is appropriate at this point to specify exactly what meaning is attributed in this work to terms ‘diffuse mode’ and ‘spot mode’. In essence, this amounts to the following question: if multiple regimes have been found in a certain current range, are there any reasons to associate one (or several) of them with diffuse mode(s) and others with spot modes?

This theoretically interesting question was addressed in [21]. The definition [21] was given in terms of distribution of the energy flux from the plasma to the cathode surface. If this distribution is non-monotonic (hollow) with the maximum not at the hottest point of the surface, then the respective solution belongs to a spot mode. In the diffuse mode, the peak heat flux density is at the hottest surface point and the energy flux distribution is monotonic. In a spot mode, the spot edge is defined as the line of the maximum heat flux density.

The above definition has already been cited in the review [22] and it is of interest to analyse it in view of results obtained in this work. There is no doubt that a distribution of the energy flux from the plasma to the cathode surface in the spot mode

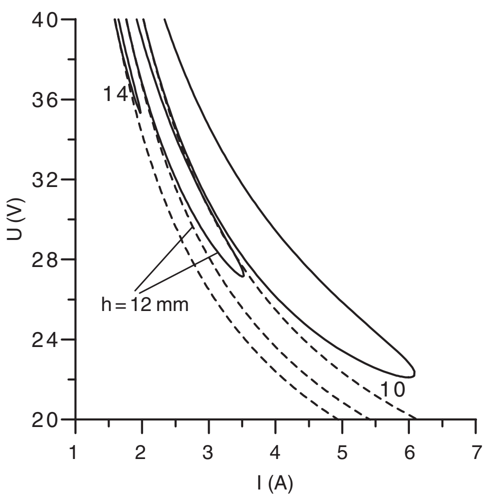

Figure 16. Current–voltage characteristic of various modes of current transfer to cathodes of different heights. $R = 0.75$ mm, argon pressure 2.6 bar. ——: spot mode. - - - -: diffuse mode.

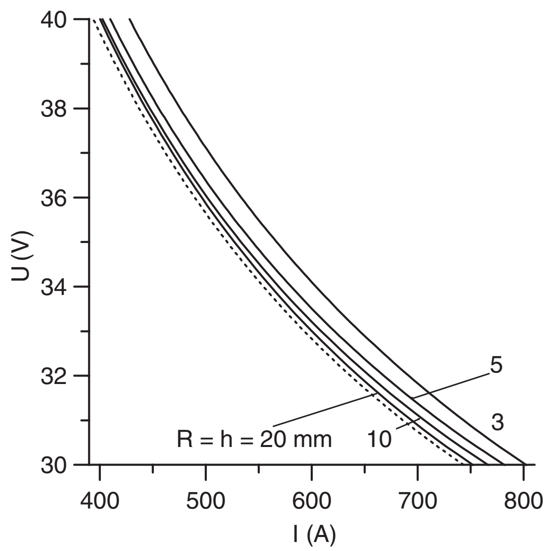

Figure 17. Current–voltage characteristic of the high-voltage mode of current transfer to cathodes of different dimensions (——). ······: solitary spot.

is non-monotonic and has a maximum on the periphery of the spot at a temperature which is lower than that at the spot centre; in fact, data on the spot mode shown in figure 5 of [21], on which the above-cited definition is based, are similar to those shown in figure 4 of [6] and in figure 5 of the present work. Nevertheless, there are some problems with this definition.

First, there is no direct relation between the maximum energy flux density occurring not at the hottest point of the surface and monotony of the energy flux density distribution. One can consider as an example states a and b which belong to the falling section of the current–voltage characteristic of the diffuse mode in figure 6: it can be seen from figures 8 and 10 that the energy flux distribution in these states is non-monotonic, however the maximum occurs at the hottest point (which is the edge of the front surface). Hence, the respective mode cannot be termed either diffuse or spot mode, if the definition [21] is used. Furthermore, states g, h, and j which belong to the rising section of the current–voltage characteristic of the diffuse mode in figure 6 should be termed belonging to the spot mode according to the definition [21], which would be hardly natural since the temperature in these states is practically constant along most of the cathode surface. These examples indicate that the definition [21] is not general enough to be applicable under the conditions of the present work.

An alternative definition may be formulated as follows. Intuitively, one means that if the current is distributed along the (front) surface of the cathode in a more or less uniform way, then the discharge operates in a diffuse mode; in a spot mode, the current distribution is essentially non-uniform. Some theorists use, instead of the term 'spot modes', the term 'modes of contracted current transfer', implying by contraction a non-uniformity of distribution of parameters along the cathode surface which is caused not only by a non-uniformity of boundary conditions (e.g. by a cathode shape). Any one of these two definitions is unambiguous in the case of a cathode in the form of a right cylinder with an insulating lateral surface, in which the temperature in the diffuse mode is exactly constant along the front surface of the cathode. Hence, one can affirm that the lines ABC and DEF in figure 11 refer to the diffuse and spot modes, respectively. A specific feature of the diffuse mode in figure 11 which can be used as a basis for a formal definition is that this is the only mode which exists at high currents. In other words, one can adopt the following simple definition: a mode which exists in a wide current range, including high currents, is a diffuse mode; modes which cease to exist as current grows are spot modes.

There is only one mode that exists at high currents in all the cases considered in this work; under the conditions of figure 6, this is the mode represented by the line ABC. In accord to the above definition, this means that lines ABC and DEF in figure 6 refer to the diffuse and spot modes, respectively, similarly to those in figure 11.

It should be emphasized that the above definition conforms to the well-known experimental fact that 'the diffuse mode is favoured by high current' (e.g. [23]); in other words, it is conventional to term diffuse a mode which occurs at high currents. It is consistent also with the above-established fact that the temperature at very high currents is constant along most of the cathode surface.

It should be emphasized also that solutions describing diffuse and spot modes are not necessarily qualitatively different at all values of the current. An example can be seen in figure 8: temperature distributions along the front surface of the cathode in the diffuse mode and on the low-voltage branch of the spot mode are not fundamentally different. Another example can be found in figure 15: there is no fundamental difference between the high-temperature region at the centre of the front surface of the cathode (which, in accord to the above definition, can be called a spot) and the high-temperature region at the cathode edge (which is not a spot and can be termed, e.g. a structure attached to the geometrical non-uniformity); in particular, any one of them may be present or absent. Hence, it is hardly possible to distinguish between diffuse and spot modes if only one current value is considered and this is why the above definition is based on a behaviour of solutions in a wide current range.

4. Concluding remarks

Solitary spots on infinite planar cathodes and diffuse and axially symmetric spot modes on finite cathodes of high-pressure arc discharges are studied in a wide current range. General features are analysed and extensive numerical results on planar and cylindrical tungsten cathodes of argon arcs are given.

The surface temperature inside a solitary spot varies relatively weakly, between a value $T_2^{(\max)}$ which corresponds to the (second) maximum of the dependence of q on T_w for a given U and a value T_2 at which q for a given U turns negative, this result being valid both in the case of low voltages (high currents) when the dependence of q on T_w at fixed U has only one maximum and in the case of high voltages (low currents) when another maximum appears. Since the temperature range $[T_2^{(\max)}, T_2]$ is relatively narrow, a reasonable estimate of the temperature inside a solitary spot may be obtained without actually solving the thermal conduction equation in the cathode body: it is sufficient to calculate the energy flux density $q(T_w, U)$ and to find $T_2^{(\max)}$ and T_2.

Asymptotic behaviour of solutions for finite cathodes in the limiting case of high currents is described by equations (8) and (9). Numerical results presented on cylindrical cathodes confirm this behaviour. These results confirm also a general pattern of current–voltage characteristics of various modes on finite cathodes suggested in [2]. A transition from the spot modes on a finite cathode in the limit of large cathode dimensions to the solitary spot mode on an infinite planar cathode has been studied and it has been found that the solitary spot mode represents a limiting form of the high-voltage spot mode on a finite cathode.

Calculations performed for a cathode with an electrically and thermally insulating lateral surface are in agreement with the predictions of the bifurcation theory [2]. On the one hand, this can be considered as a verification of the modelling approach. On the other hand, the asymptotic behaviour predicted in [2] can be conveniently used as an initial approximation for calculations of the spot-mode solution in the vicinity of the bifurcation point. Initial approximations outside this vicinity and/or for other conditions can be obtained by moving in the parameter space.

A theoretically interesting question is: if multiple regimes of current transfer have been found for a certain current range, is there an unambiguous way to associate some of them with diffuse mode(s) and others with spot modes? It was found that the following simple definition can be used (at least, under the conditions treated in this work): a mode which exists in a wide current range, including high currents, is a diffuse mode; modes which cease to exist as current grows are spot modes. It was found also that the diffuse and spot modes are not necessarily qualitatively different at every current value and this is why a definition based on a behaviour of modes in a wide current range is appropriate.

The calculated temperature of the cathode surface exceeds the melting temperature of tungsten in a number of cases, such as inside a solitary spot on an infinite planar cathode, at the centre of the front surface of a cylindrical cathode in the spot mode, at the whole front surface of the cylindrical cathode in the diffuse mode at high currents. Since the present numerical modelling has been performed for cathodes of a fixed form, the respective results should be considered as qualitative in such cases. Another factor which was disregarded in this work and which comes into play at high currents is the Joule heating in the cathode body. Additional limitations are imposed on the present results and, in particular, on the above definition of diffuse and spot modes by the thermal nature of the model considered (e.g. the spot mode may be supported by arc constriction caused by self-magnetic forces which induce the plasma jet).

It was shown analytically in [2] that two- and three-dimensional solutions describing spot modes join (or, in other terms, branch off from) the (one-dimensional) diffuse-mode solution on a cathode having the form of a right cylinder with an insulating lateral surface. In accord to this prediction, the present numerical modelling for cylindrical cathodes with an insulating lateral surface has revealed that the two-dimensional spot-mode solution joins the diffuse-mode solution. As far as cathodes with current-collecting lateral surface are concerned, the diffuse- and spot-mode solutions do not join which is not surprising since both solutions are two-dimensional, however one would expect that three-dimensional solutions describing spot modes do join two-dimensional solutions describing the diffuse mode (and maybe also spot modes) on axially symmetric cathodes. The latter question will be addressed in the third part of this work.

Acknowledgments

The work was performed within activities of the project *Theory and modelling of plasma–cathode interaction in high-pressure arc discharges* of the program POCTI of Fundação para a Ciência e a Tecnologia and FEDER and of the project *NumeLiTe* of the 5th Framework programme ENERGIE of the EC and of the action 529 of the programme COST of the EC. One of the authors (MSB) appreciates support of the Alexander von Humboldt Foundation.

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